Institut für Angewandte Mathematik

Leibniz Universität Hannover

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Program

Friday 17.06.2022

1:00-1:20 pm	Thorsten Hohage (Universität Göttingen)	Helioseismology: Mathematical foundations and algorithmic approach
1:25-1:45 pm	Maryam Parvizi (Universität Hannover)	An optimal local multilevel diagonal preconditioner for the fractional Laplacian
1:50-2:10 pm	Alexey Chernov (Universität Oldenburg)	Bayesian parameter identification of impedance boundary condition for Helmholtz problems
2:15-2:35 pm	Johannes Storn (Universität Bielefeld)	Interpolation Operator on Negative Sobolev Spaces
2:40-3:10 pm	**Coffee break**	
3.10-3.30 pm	Frank Schöpfer (Universität Oldenburg)	Certified Efficient Global Roundness Evaluation
3:35-3:55 pm	Denis Khimin (Universität Hannover)	Space-time formulation and computational studies for phase-field fracture optimal control problems
4:00-4:20 pm	Olivier Sète (Universität Greifswald)	A continuation method to find all zeros of planar harmonic mappings
4:25-4:45 pm	**Break**	
4:45-5:05 pm	Tim von Beek (Universität Göttingen)	Unfitted mixed finite element methods
5:10-5:30 pm	Utku Kaya (Universität Magdeburg)	Local pressure-corrections for incompressible flows
5:35-5:55 pm	Carolin Mehlmann (Universität Magdeburg)	The influence of solver tolerances on large scale coupled climate simulations
6:30 pm	**Conference Dinner (Broyhan Haus)**	

Saturday 18.06.2022

9:00-9:20 am	Lars Diening (Bielefeld University)	On the Sobolev stability of the L^2 projection
9:25-9:45 am	Paul Stocker (Universität Göttingen)	Embedded Trefftz discontinuous Galerkin methods
9:50-10:10 am	Hannes Uecker (Universität Oldenburg)	Global Branches in Nonlinear Schrödinger Equations with Multi–Well Potentials
10:15-10:45 am	**Coffee break**	
10:45-11:05 am	Martyna Soszynska (Universität Magdeburg)	Multirate Adaptive Time-stepping Schemes for Coupled Systems of PDEs
11:10-11:30 am	Tung Le (Universität Oldenburg)	On analytic and Gervey-class regularity for quasi-linear parametric PDE
11:35-11:55 am	Bernhard Endtmayer (Universität Hannover)	Goal oriented error estimation for nonlinear problems

Helioseismology:
Mathematical foundations and algorithmic approaches

Thorsten Hohage

University of Göttingen and Max-Planck Institute for Solar Systems Research

Helioseismology is the study of the interior of the Sun given observations of oscillations of the solar surface. Of particular interest are flows in the convection zone (the outer 30% of the Sun), but also density, pressure, and other quantities. Observations consist of the line-of-sight velocities of the solar surface obtained via Doppler shift measurements. High resolution images of oscillations of the solar surface have been recorded continuously at a cadence of 45 seconds by satellite and ground-based instruments since more than 25 years. The main bottleneck today is to extract the desired information from this huge data set. We will report on some recent progress in this direction.

We will discuss both the forward problem to predict observational data if the properties of the interior of the Sun are given, and the inverse problem to reconstruct quantities in the interior from observations on the surface. Solar oscillations are modelled as a random process driven by turbulence, and ideal noise-free data consist of the covariance operator of the surface oscillations. Computations in (local) helioseismology are typically carried out in the frequency domain. The forward problem is then defined in terms of a system of linear second order differential equations. Recently, we established well-posed of this differential equation using T-coercivity arguments and a new Helmholtz-type decomposition ([3]).

Under certain assumptions the covariance data are proportional to the imaginary part of the Green's function of the differential equations. Under this assumption and for a simplified scalar model we report on uniqueness results for several types of inverse problems, i.e. that under certain assumptions unknown interior quantities are uniquely determined by the imaginary part of Green's function at the surface ([1, 2]).

Numerical reconstructions are challenging since data are high dimensional (they depend of 5 variables, one frequency variable and 2^2 surface variables) and extremely noisy. To tackle these challenges we will discuss a new approach called iterative helioseismic holography and show some preliminary numerical results.

References

[1] A. Agaltsov, T. Hohage, and R. Novikov. Monochromatic identities for the Green function and uniqueness results for passive imaging. *SIAM J. Appl. Math.*, 78(5):2865–2890, 2018.

[2] A. D. Agaltsov, T. Hohage, and R. G. Novikov. Global uniqueness in a passive inverse problem of helioseismology. *Inverse Problems*, 36(5):055004, 21, 2020.

[3] M. Halla and T. Hohage. On the well-posedness of the damped time-harmonic Galbrun equation and the equations of stellar oscillations. *SIAM J. Math. Anal.*, 53(4):4068–4095, 2021.

An optimal local multilevel diagonal preconditioner for the fractional Laplacian

Maryam Parvizi
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In this talk, we present a local multilevel diagonal preconditioner for the integral fractional Laplacian on adaptively refined meshes generated by the finest common coarsening of a fixed mesh and a sequence of uniformly refined meshes.

The condition number of the stiffness matrix $\mathbf{A}^\ell \in \mathbb{R}^{N_\ell \times N_\ell}$ corresponding to a FEM discretization of the integral fractional Laplacian defined in $\Omega \subset \mathbb{R}^d$ using piecewise linears behaves like $\kappa(\mathbf{A}^\ell) \sim N_\ell^{2s/d} \left(\frac{h_{\max}^\ell}{h_{\min}^\ell}\right)^{d-2s}$, where $h_{\max}^\ell, h_{\min}^\ell$ denote the maximal and minimal mesh width, respectively. Considering the fact that the fractional Laplacian on bounded domains shows singularities at the boundary, the quotient $h_{\max}^\ell / h_{\min}^\ell$ is large for adaptively generated meshes. While this factor can be controlled by diagonal scaling, we also need to control the effect of the problem size. Thus, a good preconditioner controls both the impact of the variation of the element size and the problem sizes N_ℓ, simultaneously.

We show that the presented multilevel diagonal preconditioner for the fractional Laplacian on locally refined meshes leads to uniformly bounded condition number in the refinement level.

Bayesian parameter identification of impedance boundary condition for Helmholtz problems

Nick Wulbusch* Reinhild Roden† Matthias Blau† Alexey Chernov*

We address the problem of identifying the acoustic impedance of a wall surface from noisy pressure measurements in a closed room with a Bayesian approach. The room acoustics is modelled by the interior Helmholtz equation with impedance boundary conditions. The aim is to compute moments of the acoustic impedance to estimate a suitable density function of the impedance coefficient. For the computation of moments we use ratio estimators and Monte-Carlo sampling. We consider two different experimental scenarios. In the first scenario the noisy measurements correspond to a wall modelled by impedance boundary conditions. In this case the Bayesian algorithm uses a model that is (up to the noise) consistent with the measurements and therefore reaches very good accuracy observed in our numerical experiments. In the second scenario the noisy measurements come from a coupled acoustic-structural problem corresponding to the case of a wall made of glass, whereas the Bayesian algorithm still uses a model with impedance boundary condition. In this case the parameter identification model is inconsistent with the measurements and therefore is not capable to represent them well. Nonetheless, for particular frequency bands the Bayesian algorithm identifies estimates with relatively high likelihood. Outside these frequency bands the algorithm fails. We discuss the results of both examples and possible reasons for the failure of the latter case for particular frequency values.

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INTERPOLATION OPERATOR ON NEGATIVE SOBOLEV SPACES

L. DIENING, J. STORN, T. TSCHERPEL

ABSTRACT

This talk summarizes a joint project with Lars Diening and Tabea Tscherpel (Bielefeld University), where we introduce a Scott–Zhang type projection operator Π mapping to Lagrange elements of arbitrary polynomial order. In addition to the usual properties, the operator Π is stable in the $H^{-1}(\Omega)$ norm and allows for optimal rates of convergence. Moreover, we discuss alternative operators with similar properties.

The novel operator Π allows us to design a local interpolation operator for semi-discretizations and tensor-product spaces for parabolic problems with optimal rates of convergence. Moreover, it allows us to smoothen rough right-hand sides in least-squares finite element methods which leads to quasi-optimality with respect to the energy norm plus some higher-order data-approximation error.

Certified Efficient Global Roundness Evaluation

Frank Schöpfer and Alexey Chernov

Abstract

In metrology applications an object is considered round enough, if all measurement points on its surface lie between two spheres (circles) around a common center, such that the difference of their radii is smaller than some prescribed tolerance. Mathematically, the *roundness* of any set $S \subset \mathbb{R}^d$ may be defined as the smallest possible difference of radii for a suitable center, and its computation leads to a nonconvex optimization problem.

Here we discuss a global search algorithm for the solution of the roundness problem, whose formulation is valid in any dimension, and covers both finite sets of data points as well as infinite sets like polygonal chains or triangulations of surfaces. We give theoretical estimates for the cost to reach a desired accuracy, and validate the performance of the algorithm with numerical experiments.

The talk is based on [Schöpfer, F., Chernov, A. Certified Efficient Global Roundness Evaluation. J Optim Theory Appl 186, 169–190 (2020)]

Space-time formulation and computational studies for phase-field fracture optimal control problems

Denis Khimin[1], Marc C. Steinbach[2], Thomas Wick[3]
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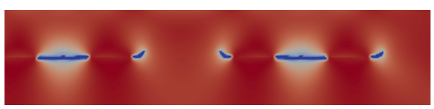
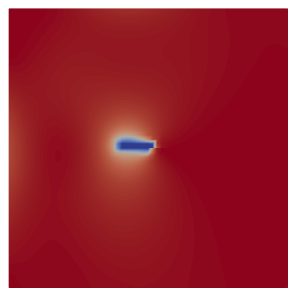
In this presentation we provide a space-time discretization scheme for optimal control problems involving phase-field fracture as a constraint. The forward problem is described by a energy minimization problem and the corresponding Euler-Lagrange-equations. First a dicontinuous Galerkin formulation for the state equation is derived including the crack ireversibility constraint, i.e. an inequality constraint in time. The optimal control setting is formulated by a reduced approach using the Lagrangian for a tracking type functional. We conclude the talk with some numerical experiments substantiating the overall algorithm.





Acknowledgement
This work has been supported by the German Research Foundation, Priority Program 1962 (DFG SPP 1962) with in the subproject *Optimizing Fracture Propagation using a Phase-Field Approach* with the project number 314067056 and by the DFG – SFB1463 – 434502799.

References

[1] I. Neitzel and T. Wick and W. Wollner; An Optimal Control Problem Governed by a Regularized Phase-Field Fracture Propagation Model. *SIAM Journal on Control and Optimization* (2017).

[2] D. Khimin, M. C. Steinbach, and T. Wick. Space-time formulation and time discretization of phase-field fracture optimal control problems, 2021.

A continuation method to find all zeros of planar harmonic mappings

Olivier Sète (Universität Greifswald)

May 30, 2022

We present a continuation method to compute all zeros of a nondegenerate harmonic mapping f (complex-valued harmonic function) in the plane. Our method works without any prior knowledge of the number of zeros or their approximate location. We start by computing all solutions of $f(z) = \eta$ with $|\eta|$ sufficiently large and then track all solutions as η tends to 0 to finally obtain all zeros of f. Using theoretical results on harmonic mappings we analyze where and how the number of solutions of $f(z) = \eta$ changes and incorporate this into the method. This allows us to overcome typical difficulties when using continuation methods. In our numerical examples the method always terminates with the correct number of zeros, is very fast, and is highly accurate in terms of the residual.

This talk is based on joint work with Jan Zur (Technische Universität Berlin).

References

[1] O. Sète and J. Zur, *A Newton method for harmonic mappings in the plane*, IMA J. Numer. Anal., 40 (2020), pp. 2777–2801.

[2] O. Sète and J. Zur, *Number and location of pre-images under harmonic mappings in the plane*, Ann. Fenn. Math., 46 (2021), pp. 225–247.

[3] O. Sète and J. Zur, *The transport of images method: computing all zeros of harmonic mappings by continuation*, IMA J. Numer. Anal., 2021.

Unfitted mixed finite element methods

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Keywords: *unfitted FEM, mixed FEM, stabilization, mass conservation*

Geometrically unfitted finite element methods such as CutFEM, Finite Cell, XFEM or unfitted DG methods have been developed and applied successfully in the last decades to a variety of problems ranging from scalar PDEs on stationary domains to systems of PDEs on moving domains and PDEs on level set surfaces. Two key techniques that enabled the success of these methods for a broad range of applications are

- the weak imposition of boundary conditions, e.g. via Nitsche method and
- stabilization techniques to deal with bad cuts such as ghost penalties or aggregations

These approaches combined with established tools of finite element methods allowed to apply and analyze unfitted methods in many fields. An important and powerful class of finite elements are mixed methods based on special vectorial finite elements such as $H(\text{div})$-conforming spaces. These are typically tailored to preserve conservation properties like mass conservation exactly in the discretization. A basic example is the mixed formulation of the Poisson problem which in the fitted case takes the following form:

Strong form:

Find σ, u with $u = 0$ on $\partial\Omega$, s.t.

$$\sigma - \nabla u = 0 \quad \text{in } \Omega, \qquad \text{(F)}$$

$$\operatorname{div} \sigma = -f \quad \text{in } \Omega, \qquad \text{(C)}$$

Mixed FEM:

Find $\sigma_h \in \Sigma_h \subset H(\text{div}, \Omega)$, $u_h \in Q_h \subset L^2(\Omega)$, s.t.

$$(\sigma_h, \tau_h)_\Omega + (\operatorname{div} \tau_h, u_h)_\Omega = 0 \text{ for all } \tau_h \in \Sigma_h, \qquad (F_h)$$

$$(\operatorname{div} \sigma_h, v_h)_\Omega = (-f, v_h)_\Omega \text{ for all } v_h \in Q_h. \qquad (C_h)$$

When looking for unfitted versions of these type of discretizations a major difficulty is stability, especially stability in the sense of the Ladyzhenskaya–Babuška–Brezzi (LBB) condition. An LBB condition that may be valid on fitted meshes for a given pair of spaces Σ_h and Q_h w.r.t. the background mesh can easily degenerate on arbitrary cuts. Additional stabilizations like the ghost penalty method which introduces couplings between u_h and v_h have been used in the literature to arrive at stable unfitted mixed discretizations. However, these stabilizations perturb the exact conservation property. In this talk we introduce a new approach that yields an LBB-stable discretization of the unfitted mixed problem without the need for stabilizations of ghost penalty type that pollute the mass balance.

Local pressure-corrections for incompressible flows

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This is a joint work with Thomas Richter *(Otto-von-Guericke-Universität Magdeburg)* and Malte Braack *(Christian-Albrechts-Universität zu Kiel)*.

Pressure-correction methods facilitate approximations of solutions to time-dependent incompressible fluid flows by decoupling the momentum equation from the continuity equation [1]. A common strategy used by several pressure-correction methods is:

- compute a (not necessarily divergence-free) predictor velocity field,
- solve a Poisson problem for the pressure,
- project the predictor velocity field onto a divergence-free one.

In cases, where an explicit time-stepping scheme for the momentum equation is employed, the Poisson problem for the pressure remains to be the most expensive step. We here present a domain decomposition method that replaces the pressure Poisson problem from step (ii) with local pressure Poisson problems on non-overlapping subregions [2, 3]. No communication between the subregions is needed, thus the method is favorable for parallel computing. We illustrate the effectivity of the method via numerical results.

References

[1] J.L. Guermond, P. Minev, J. Shen, *An overview of projection methods for incompressible flows.* Comput. Methods Appl. Mech. Engrg. **195**:44-47 (2006), 6011-6045.

[2] M. Braack, U. Kaya, *Local pressure-correction for the Stokes system.* J. Comput. Math. **38**:1 (2020), 125-141.

[3] U. Kaya, R. Becker, M. Braack, *Local pressure-correction for the Navier-Stokes equations.* Internat. J. Numer. Methods Fluids. **93**:4 (2021), 1199-1212.

The influence of solver tolerances on large scale coupled climate simulations

C. Mehlmann Y. Shih, G. Stadler, L. Ramme, P. Korn

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Subject of the talk is the influence of numerical tolerances as well the development of a new Newton solver for a specific problem in climate science, the Snowball Earth hypothesis: 630-750 million years ago the Earth might have been in deep freeze with completely ice-covered oceans. Climate simulations that investigate the Snowball Earth hypothesis typically approximate the nonlinear sea ice processes with only a few solver iterations due to the high numerical costs. The influence of this inaccuracy as well as the development of fast and robust solving methods are current research questions.

We show that the underlying coupled set of equations that describes the sea ice dynamics in climate models can be formulated as an energy minimization problem. Based on the theoretical analysis we derive a new Newton method that leads to faster and more robust Newton convergence than currently used methods. In the context of the Snowball Earth hypothesis, we demonstrate that the numerical tolerances are more important for the resulting climate dynamics than usual tuning parameters. These are alarming results, since the numerical accuracy of nonlinear sea ice processes have never been considered to be important for past, present, or future climate projections.

References

[1] C. Mehlmann, P Korn, Sea-ice dynamics on triangular grids, J. Comput. Phys., 2021

[2] Y. Shih, C. Mehlmann, M. Losch, G. Stadler Robust and efficient primal-dual Newton-Krylov solvers for viscous-plastic sea-ice models, arxiv.org/abs/2204.10822, 2022

Lars Diening (Bielefeld University)

On the Sobolev stability of the L^2 projection

The L^2-projection to the space of Lagrange finite elements is a fundamental tool in numerical analysis. It is in particular of importance for the study of parabolic problems. To analyze these problems it is necessary to use the stability of the L^2-projection with respect to the Sobolev space $W^{1,2}$. Indeed, it was shown recently that the equivalence of the discretization error and the best approximation error in terms of function spaces is equivalent to the Sobolev stability of the L^2-projection.
The Sobolev stability is easily proved for quasi-uniform meshes, but is very difficult to obtain for locally refined meshes. In this talk I will present recent results on the Sobolev stability of the L^2-projection in 2D and 3D for arbitrary degree of the polynomials. This is a joint work with Johannes Storn and Tabea Tscherpel from Bielefeld University (Germany).

Embedded Trefftz discontinuous Galerkin methods

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Abstract

In Trefftz discontinuous Galerkin methods a partial differential equation (PDE) is discretized using discontinuous shape functions that are chosen to be elementwise in the kernel of the corresponding differential operator. We propose a new variant, the embedded Trefftz discontinuous Galerkin method, which is the Galerkin projection of an underlying discontinuous Galerkin (DG) method onto a subspace of Trefftz-type. The subspace can be described in a very general way and to obtain it no Trefftz functions have to be calculated explicitly, instead the corresponding embedding operator is constructed. In the simplest cases the method recovers established Trefftz-DG methods. But the approach allows to conveniently extend to general cases, including inhomogeneous sources and non-constant coefficient differential operators.

The method The embedded Trefftz method, presented in [1], side-steps the explicit construction of a Trefftz space and instead uses the corresponding embedding, which is easily constructed. In its simplest form, the method can be seen as a convenient way to set up the linear system of a Trefftz-DG discretization by means of a Galerkin projection of a standard DG method onto its Trefftz subspace. Instead of implementing Trefftz functions explicitly, the characterization of the Trefftz function space as the kernel of an associated differential operator is exploited to construct an embedding of a Trefftz subspace into the DG space in a very generic way. We denote this embedding as the *Trefftz embedding*. The construction requires only element-local operations of small matrices and can be done in parallel. To compute the discrete kernel numerically several established methods exists, e.g. one can use QR factorizations, singular, or eigenvalue decompositions. We denote this approach as *embedded Trefftz DG method*.

Novel features The generic way the linear systems are set up allows to apply the concept of Trefftz methods beyond their previous limitations. Two of these limitations that can be conveniently exceeded are:

1. Inhomogeneous equations where the solution is not exactly in the kernel of a differential operator, but affinely shifted, are difficult to handle for most Trefftz methods. Only for specific cases special solutions have been designed in the literature to overcome the limitation to homogeneous equations. Exploiting that the embedded Trefftz DG method is build on an underlying DG method homogenization becomes very convenient to realize for the embedded Trefftz method.

2. An embedding of a meaningful Trefftz-type subspace of the DG space can easily be constructed even when a standard Trefftz space is out of reach. A standard Trefftz space may already be missing in two important cases. Firstly, if differential operators with non-uniform derivatives (in at least one direction) are involved - as e.g. in Helmholtz or convection-diffusion equations - there are no polynomial solutions to the PDE. Second, if coefficient are involved in the differential operators which are not piecewise-constant there are typically also no polynomial solutions to the PDE. Hence, in both cases there will be no meaningful Trefftz space. However, reasonable *weak Trefftz subspaces* can easily be setup which share the advantages of traditional Trefftz spaces.

References

[1] C. Lehrenfeld and P. Stocker, *Embedded Trefftz discontinuous Galerkin methods*, arXiv preprint, arXiv:2201.07041, (2022).

Global Branches in Nonlinear Schrödinger Equations with Multi–Well Potentials

Hannes Uecker (Oldenburg), Panos Kevrekidis (Amherst), Eduard Kirr (Urbana)

From the perspective of global bifurcation theory, we study solutions of nonlinear Schrödinger (or Gross–Pitaevskii) models $\mathrm{i}u_t = -\Delta u + V(x)u + \sigma|u|^{2p}u$, where $u(x,t) : \mathbb{R}^n \times \mathbb{R} \to \mathbb{C}$, with Δ the Laplacian, focusing ($\sigma < 0$) nonlinearity of power $p > 0$, and where $V(x) : \mathbb{R}^n \to \mathbb{R}$ is an external real potential, for instance of multi–well type. We look for standing wave solutions $u(t,x) = \mathrm{e}^{iEt}\psi(x)$, and first establish new results in the regime of large (asymptotically infinite) eigenvalue parameter E, namely the existence of multi–peak ground states localizing at maxima or saddles, while near minima only single peaks may localize. Subsequently we discuss implications of this limiting behavior for branches at finite E, and illustrate the results numerically, in 1D and 2D, in very good quantitative agreement with the analytical results. Additionally, we explore some reconnection phenomena for double–well potentials as the inter–well distance changes. Our computations illustrate that these features are generic, i.e., arise for different potentials and different values of the nonlinearity exponent p. The dynamics near the newly identified waveforms are also explored via direct numerical simulations.

Multirate Adaptive Time-stepping Schemes for Coupled Systems of PDEs

Martyna Soszyńska[1], Thomas Richter[1]

[1] Otto-von-Guericke-Universität Magdeburg

We study time discretization schemes for coupled systems of partial differential equations. We assume that the subproblems are defined over spatially distinct domains with a common interface, where the coupling is enforced. Each of the physical problems can be governed by a different type of equations (either parabolic or hyperbolic) and therefore can exhibit different dynamics. Fluid-structure interactions are one type of important application problems that fall into this framework. Here however, we will consider linear problems only.

Our aim is to develop time discretization schemes allowing for different time-step sizes in each of the domains without violating the coupling conditions. We are able to achieve it by formulating the problems within the space-time framework. Although the formulation is monolithic, we solve the systems sequentially relying on a partitioned approach. We further develop an a posteriori error estimator based on the dual weighted residual method. This estimator is then used as an adaptivity criterion [1]. To justify this method, we show stability estimates.

References

[1] M. Soszyńska, T. Richter. Adaptive time-step control for a monolithic multirate scheme coupling the heat and wave equation. *BIT Numerical Mathematics*, Vol. 61, 2021.

On analytic and Gervey-class regularity for quasi-linear parametric PDE

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We investigate a class of parametric quasi-linear PDEs with homogeneous essential boundary conditions where the forcing term, coefficients (also the solution u) may depend on a parameter . For every $\varepsilon > 0$, we build an approximate $u_\varepsilon()$, solution of the regularized PDE belongs to a subset of $W^{1,\infty}(D)$, such that $u - u_\varepsilon < \varepsilon$ for almost all values of the parameter . Based on the implicit function theorem, we establish under appropriate additional assumptions both analytic and Gervey-class regularities of the parameter-to-solution map $\mapsto u_\varepsilon()$. This result has immediate implications for convergence of various numerical schemes for quasi-linear PDEs under uncertainty, in particular, for quasi-linear PDEs with random forcing term.

Goal oriented error estimation for nonlinear problems

Endtmayer Bernhard[1,3], Ulrich Langer[2], Andreas Schafelner[2], and Thomas Wick[1,3]

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In this talk, we derive goal oriented a posteriori error estimates for nonlinear problems using the dual-weighted residual (DWR) method. These results hold true for both nonlinear partial differential equations and nonlinear functionals of interest. Our theoretical findings and algorithmic developments are substantiated with some numerical tests.