Institut für Angewandte Mathematik

Leibniz Universität Hannover

Institut für Angewandte Mathematik
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Oberseminar
Analysis und Theoretische Physik

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Universität Magdeburg

"Biharmonic Green functions: New examples for sign change and estimates from above and below"

Abstract:

The Green function $G_{-\Delta,\Omega}$ for the Laplacian under Dirichlet boundary conditions in a bounded smooth domain $\Omega \subset \mathbb{R}^n$ enjoys in dimensions $n \geq 3$ the estimate:

$0 \leq G_{-\Delta,\Omega}(x,y) \leq \frac{1}{n(n-2)e_n}|x-y|^{2-n}$. Here, e_n denotes the volume of the unit ball $B = B_1(0) \subset \mathbb{R}^n$. This estimate follows from the maximum principle, the construction of $G_{-\Delta,\Omega}$ and the explicit expression of a suitable fundamental solution. In higher order elliptic equations the maximum principle fails and deducing Green function estimates becomes an intricate subject. We consider the clamped plate boundary value problem as a prototype:

$$\begin{cases} \Delta^2 u = f & \text{in } \Omega, \\ u = |\nabla_u| = 0 & \text{on } \partial\Omega \end{cases}$$

First, I shall explain recent examples for bounded smooth domains where one has a sign changing solution even for constant right hand side $f(x) \equiv 1$. Next, I shall discuss estimates for the corresponding Green function $G_{\Delta^2,\Omega}$ focusing on two aspects:

- Keeping Ω fixed, can one show – although $G_{\Delta^2,\Omega}$ is in general sign changing – that it is somehow "almost positive"?
- Removing arbitrarily small holes (with almost infinite curvature) from a fixed domain Ω prevents uniform constants in classical Green function estimates. Can one nevertheless deduce estimates for this singular family of domains which are uniform with respect to the size of the hole?

The lecture is based on joint works with F. Robert (Nancy) and G. Sweers (Cologne).

Dienstag, 04.02.2014, 15:00 Uhr, Raum g005
Hauptgebäude der Universität

Über Ihren Besuch würden sich freuen:

Prof. Dr. Joachim Escher
Prof. Dr. Olaf Lechtenfeld
Prof. Dr. Elmar Schrohe
Prof. Dr. Christoph Walker