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“A variational approach to a class of nonlocal evolution equations and existence of solitary waves of the Whitham equation”

Abstract:
We prove the existence of solitary-wave solutions for a class of nonlocal evolution equations of the form \( u_t + \{n(u) + Lu\}_x = 0 \). The linear operator \( L \) is nonlocal of negative order, whereas the nonlinearity \( n \) is local and of superlinear growth near the origin. Using the methods of minimization-penalisation and concentration-compactness we find periodic solutions converging to conditionally stable, smooth minimisers of small amplitude.

Our analysis includes the case of the Whitham equation, the linear terms of which match the dispersion relation for gravity water waves on finite depth. The Whitham equation has a global bifurcation branch of \( 2\pi \)-periodic, smooth, traveling-wave solutions and is conjectured to admit a “highest”, cusped, wave. For the solitary case, we show that the Whitham minimisers approximate the KdV-solitons in the small-amplitude limit.

The talk is based on ongoing work with Mark Groves, Saarbrücken, and Erik Wahlén, Lund, as well as joint work with Henrik Kalisch, Bergen.