Spectral flow and the Riesz continuity of the Atiyah–Singer Dirac operator under bounded perturbations of the metric or local boundary conditions

The study of the spectral flow was initiated by Atiyah and Singer in 1969 by considering the case of bounded self-adjoint Fredholm operators. It is a topic of importance, particularly given its connections and relevance to particle physics. This also necessitates its study in the setting of unbounded operators. In this case, a particular choice of topology has to be made, and the so-called Riesz topology is the favoured one given that it naturally connects to the topological point of view taken in the original work of Atiyah and Singer. The motivations for our work comes from spectral flow questions surrounding the Atiyah-Singer Dirac operator on smooth Riemannian Spin manifolds with smooth compact boundary. Our geometric assumptions are that the metric is complete with uniform lower bounds on injectivity radius along with bounds on the Ricci curvature and its first derivatives. We demonstrate that this operator is stable in the Riesz topology under two different settings: (1) bounded perturbations of the metric in the case the manifold has no boundary, and (2) bounded perturbations of local boundary conditions when the boundary is nonempty. The significance of our results is that we do not require smallness in first derivatives of the metric or of the boundary conditions in establishing continuity. Moreover, we allow for our manifolds to be non-compact and we do not assume that the operators are Fredholm. These results are obtained by obtaining similar results for a more wider class of elliptic first-order differential operators on vector bundles satisfying certain general curvature conditions. At the heart of our proofs lie methods from Calderón-Zygmund harmonic analysis coupled with the modern operator theory point of view developed in proof of the Kato square root conjecture.