Robust Preconditioned Iterative Solvers for Discretized Reduced Optimality Systems

by
Ulrich Langer
(Johannes Kepler University Linz)

We propose, analyze, and test new robust iterative solvers for systems of linear algebraic equations arising from the finite element discretization of reduced optimality systems defining the finite element approximations to the solution of elliptic, parabolic, and hyperbolic distributed optimal control problems with both the standard L_2 and the more general energy regularizations. In contrast to the usual time-stepping approach, we discretize the optimality system arising from time-dependent optimal control problems by space-time continuous piecewise-linear finite element methods on fully unstructured simplicial meshes in the same fashion as in the case of elliptic problems.

If we aim at the best approximation of the given desired state y_d by the computed finite element state y_h, then the optimal choice of the regularization parameter ϱ is linked to the mesh-size h by the relations $\varrho = h^4$ and $\varrho = h^2$ for the L_2 and the energy regularization, respectively. For this setting, we can construct robust (parallel) iterative solvers for the reduced finite element optimality systems. These results can be generalized to variable regularization parameters adapted to the local behavior of the mesh-size that can heavily change in case of adaptive mesh refinement. In practice, the solver should be embedded in a nested iteration procedure that starts from some suitable coarse mesh and proceeds with finer and finer (adaptive) meshes until the desired accuracy of the approximation of y_d is reached or the costs for the control exceed a prescribed threshold. The numerical results confirm the theoretical findings.

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