Chapter 2

Physical and technical fundamentals of gas networks

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Abstract *This chapter describes the fundamentals of gas transport. This includes an introduction to the basic terminology and basic physical laws with respect to natural gas. Then the basic elements needed to represent gas networks are discussed: pipes, resistors, valves, control valves, and compressor machines and drives. These elements can be grouped into larger entities like compressor groups and subnetwork operation modes.*

Natural gas is distributed through large and complex pipeline networks; for instance, the total length of the European network is more than 100 000 km. The main part of the network consists of interconnected gas pipelines. The gas flow is controlled or affected by additional components like compressors, (control) valves, and resistors. The goal of this chapter is to describe these components as well as their physical and technical properties. See also the tables inside the front and back covers of this book for a list of the physical/technical quantities and constants introduced in this chapter, respectively.

The contents of this chapter are taken from many sources in the literature and are well known. For general literature concerning the description of gas networks, we refer the reader to the books by Cerbe (2008), Lurie (2008), Osiadacz (1987), and Zucker and Biblarz (2002), as well as the article by Fasold and Wahle (1992). Concerning the modeling of compressor machines, see Odom and Muster (2009) for a recent survey.

Let us add a general comment on the literature in this area: The formulas on gas transport and their derivations that are used in both theory and practice are sometimes not derived in scientific literature, but appear in documentation of simulation software or working papers, e.g., of the Pipeline Simulation Interest Group (PSIG). Moreover, some literature, in particular, more technical articles, is not available in English. Whenever possible, we (also) cite an English article. There are, however, cases for which we do not know any English source and have to refer to German literature.

Note that certain parts that we describe in the following are *virtual* components of the network (e.g., resistors) and do not directly represent physical entities. As such they are based on modeling *decisions*—in contrast to plain descriptions of physical laws. Moreover, all given formulas represent only an abstraction of reality, since certain practical effects are not included. This is clearly motivated by the trade-off between an accurate description of reality and the possibility of solving the resulting models. Furthermore, note that our presentation is influenced by the goal to treat the stationary case.

This chapter is structured as follows: Section 2.1 gives a short introduction into the basic concepts of gas transport from a technical point of view. In Section 2.2, we present in detail the properties of different kinds of natural gas before we give a description of each type of network element: pipes (Section 2.3.1), resistors (Section 2.3.2), valves (Section 2.3.3), control valves (Section 2.3.4), and compressor machines and drives (Section 2.3.5). These different types of network elements can be combined into larger entities: control valve stations (Section 2.4.1), compressor groups (Section 2.4.2), and subnetworks involving subnetwork operation modes (Section 2.4.3) such as compressor stations. Finally, Section 2.5 introduces the graph representation for gas transport networks.

Before we begin, let us remark that we generally use SI (base) units in this book, while different units are used throughout the gas literature (e.g., the old *barg*, which measures pressure in bar, zero-referenced to the atmospheric pressure).

2.1 ▪ Gas transport

Gas is fed into the network at certain points, so-called entries, and may be withdrawn from the network at other points, so-called exits. The terms *entry* and *exit* abstract from reality, in the sense that an entry may be, e.g., a natural gas field as well as an interconnection point from another gas network. Also the term exit may refer to an interconnection point as well as to a certain customer such as a factory or a gas power plant.

The network operator is responsible for the successful transport of gas, while its customers are allowed to nominate any amount of gas per time unit at the entries and exits subject to contractual and legal constraints. To this end, a nomination is issued in terms of thermal power, supplied at a certain set of entries and withdrawn at a certain group of exits. In addition, the customers are legally bound to issue only *balanced nominations*, meaning that the total power nominated at entries must be equal to the total power nominated at exits. Moreover, contracts normally require a minimal gas pressure at exits. In contrast to gas consumers and natural gas fields, certain points in the network are *hybrid points*, i.e., they can be used bidirectionally and therefore must be treated as exits at one point of time and must be considered entries at another point of time. Examples are network interconnection points and gas storages. In situations where a gas storage is filled, it must be considered an exit, while it takes the role of an entry when gas is withdrawn from the storage. Similarly interconnection points change from entries to exits and vice versa depending on the flow directions.

In order to facilitate a successful gas transport, gas flow, pressure, and composition have to be controlled using the components of the network. For example, pipes in regional subnetworks are designed for rather low pressure levels, while large transport pipelines can be operated at high pressure. So-called control valves allow reducing pressure from high to low pressure pipelines at transition points. Moreover, when gas is transported through a pipeline network, it experiences a pressure drop. In order to transport gas over large distances, pressure has to be increased at so-called compressors. Furthermore, a network contains a number of valves, which might be open or closed, and which allow routing the gas through the network.

In the following, valves, control valves, and compressor groups are called *controllable* or *active* network elements. All other elements are called *passive*. Besides pipes, a gas network usually contains a number of further passive elements, e.g., filtering or measuring stations.

The gas network is usually remotely controlled by human dispatchers. From a central operating room they continuously monitor the status of the network and make decisions according to the current and forecast flow situation. They control the settings of active elements under regular operational conditions. In case of sudden emergencies, such as an overheating of gas or a loss of pressure due to a pipeline burst or leakage, an automated pipeline shut-down in the affected parts of the network limits the impact of such rare events. The task of network operators is to maintain a stable flow of gas through the network between the entries and exits such that this flow does not violate technical restrictions such as maximum pressure levels of pipelines, or minimum entry pressure levels at compressor groups, for example. Furthermore, pressure levels and the amount of flow have to be within a specified interval at the entries and exits due to contractual requirements. Operating networks within these limits should be achieved using a minimal amount of energy that is consumed, mainly by compressors.

Dispatching is performed dynamically over time, i.e., transient gas flows are considered. For long-term planning, which is the focus of this book, it is also interesting to consider steady state or stationary states. See Section 5.3.1 for a discussion of this.

2.2 ▪ Gas properties

One main component to describe the behavior of gas is the so-called *equation of state* connecting *mass*, *pressure* p, *volume* V, and *temperature* T. These quantities are sufficient to determine the inner state of a simple thermodynamical system for gas (i.e., systems with a single homogeneous gas mixture). The latter three are known as thermal state quantities. A state quantity is a property of a system that depends only on the current state of the system and not on the process leading to that state. There are two types of state quantities: inner and outer states. The investigation of the outer state (e.g., position, velocity) is the purpose of mechanics, as described in the following sections for each network element. Thermodynamics deals with the inner state, i.e., the equation of state.

For an *ideal gas*, the so-called *equation of state for ideal gases* or, in short, *ideal gas law* (see, e.g., Menon (2005)) applies:

$$p\,V = \tilde{n}\,R\,T = N\,k_B\,T,$$

where V denotes the gas volume in m^3, $\tilde{n}$ is the *amount of substance* of the gas in mol, and $R = 8.3144621$ J/mol/K $= N_A\,k_B$ is the *universal gas constant*. Here, the *Avogadro constant* is denoted by $N_A = 6.02214129 \times 10^{23}$/mol and $k_B = 1.3806488 \times 10^{-23}$ J/K denotes the *Boltzmann constant*. The *number of particles* is given by $N = \tilde{n}\,N_A$. The assumptions for an ideal gas are that molecules in the gas are point-like and do not interact with each other.

A *real gas*, however, shows a different relation between pressure, temperature, and volume than ideal gases, especially because of interaction between the gas particles. This resulting deviation of a real gas from an ideal gas is described by the so-called *compressibility factor*, which is defined as

$$z := \frac{p\,V}{\tilde{n}\,R\,T} = \frac{p\,V}{N\,k_B\,T}.$$

In order to derive an equation of state for a real gas from the ideal gas law, one can then apply the so-called *virial expansion* (see Plischke and Bergersen (2006) or Hill (1960)), which is the following power series in N/V:

$$z = 1 + \sum_{i=1}^{\infty} B_{i+1}(T) \left(\frac{N}{V}\right)^{i}. \tag{2.1}$$

The coefficient B_i is called the ith *virial coefficient*. One of the main benefits of the virial expansion is that $z = z(p, V, N, T)$ is written as a function that depends only on N/V and T, i.e., $z = z(N/V, T)$. Equivalently, the compressibility factor can be formulated as a power series in p, and hence in a form $z = z(p, T)$ that depends only on p and T. Such a dependency,

$$z = 1 + \sum_{i=1}^{\infty} \tilde{B}_{i+1}(T)\, p^{i}, \tag{2.2}$$

is more adequate for our purposes and is assumed throughout this book. Note that the compressibility factor z is equal to 1 for an ideal gas. Moreover, the virial expansion can be used to derive arbitrarily many approximations of the compressibility factor, i.e., of state equations, by truncating higher order terms in (2.1) or (2.2). With the abstract introduction of the compressibility factor z as above, the relationship

$$p\, V = \tilde{n}\, R\, T\, z = N\, k_{\mathrm{B}}\, T\, z, \tag{2.3}$$

in terms of the thermal state quantities p, V, and T, is known as the *thermodynamical standard equation of state for real gases* Starling and Savidge (1992).

In practice, the computation of z must be performed approximately. The most accurate and most general approximation of the compressibility factor known is given by the GERG-2008 equation of state Kunz and Wagner (2012). Other high accuracy approximations, at least for the case of natural gas transport through pipeline networks, are constituted by the AGA-8 DC-92 equation Starling and Savidge (1992) and the GERG-2004 equation Kunz et al. (2007). These three equations can be derived from the virial expansion (2.1) (or (2.2)) and thus from the laws of statistical thermodynamics. These formulas require detailed knowledge about the composition of gas. However, there are also a number of empirical formulas for the compressibility factor that depend only on the *reduced pressure* $p_{\mathrm{r}} = p/p_{\mathrm{c}}$ and the *reduced temperature* $T_{\mathrm{r}} = T/T_{\mathrm{c}}$ of a gas mixture. Here, p_{c} denotes the *pseudocritical pressure* and T_{c} is the *pseudocritical temperature* (measured in Pa and K, respectively). Below or at the so-called *critical temperature*, a pure gas may be liquefied under pressure. Above the critical temperature, this is impossible independent of the pressure. Based on this temperature, the *critical pressure* is defined as the minimum pressure which would suffice to liquefy a pure gas at its critical temperature. For a gas mixture, critical temperature and critical pressure are called pseudocritical. When approaching the *pseudocritical point* $(p_{\mathrm{c}}, T_{\mathrm{c}})$ in a pressure-temperature phase diagram, differences between the two aggregate phases vanish, i.e., properties of the gaseous and liquid phases (e.g., density) are indistinguishable and can no longer be regarded as being different phases. Instead, above this point the substance is in a homogeneous supercritical phase.

Two common formulas that are used to compute the compressibility factor z and are purely based on the reduced pressure and temperature of a gas mixture are the formula of Papay (see Papay (1968) and Saleh (2002), Chap. 2),

$$z(p, T) = 1 - 3.52\, p_{\mathrm{r}}\, e^{-2.26\, T_{\mathrm{r}}} + 0.247\, p_{\mathrm{r}}^{2}\, e^{-1.878\, T_{\mathrm{r}}}, \tag{2.4}$$

and an equation from the American Gas Association (AGA) (see Králik et al. (1988)):

$$z(p, T) = 1 + 0.257\, p_{\mathrm{r}} - 0.533\, \frac{p_{\mathrm{r}}}{T_{\mathrm{r}}}. \tag{2.5}$$

The Papay equation is known to be accurate up to 150 bar, whereas the AGA equation is known to be accurate only up to 70 bar (see LIWACOM (2004)).

Finally, besides the virial equation of state (2.1) and the thermodynamical standard equation of state (2.3), several other empirical equations with different ranges of validity exist. Among those, the cubic equations of state of Waals (1873), Benedict, Webb, and Rubin (1940), Redlich and Kwong (1949), Soave (1972), Peng and Robinson (1976), Stryjek and Vera (1986), and Stryjek and Vera (1986) are commonly used in engineering (see, e.g., Rao (2003)). In the remainder of this book we restrict ourselves to the application of the thermodynamical standard equation of state for real gases (2.3) if not stated otherwise. Hence, all laws and formulas involving thermodynamics are based on this fundamental relationship and might differ if an alternative equation of state is desired. Nonetheless, most equations of state can be rewritten in a form compatible with Eq. (2.3) and Eq. (2.1).

Another important quantity in thermodynamics is the *specific isobaric heat capacity* c_p. It measures the amount of energy required to raise the temperature of one kilogram of gas by one Kelvin at constant pressure. For our purposes, the introduction of the related *molar isobaric heat capacity* $\tilde{c}_p = c_p m$, where m denotes the *molar mass*, is convenient. For real gases, the specific or molar isobaric heat capacity is typically split up into an ideal gas term and a real gas correction:

$$c_p(p,T) = \frac{\tilde{c}_p(p,T)}{m} = \frac{1}{m}\left(\tilde{c}_p^0(T) + \Delta\tilde{c}_p(p,T)\right).$$

Here, $\tilde{c}_p^0$ is the molar heat capacity of ideal gas, $\Delta\tilde{c}_p$ is a correction for real gas, and m is the molar mass. The ideal gas term

$$\tilde{c}_p^0(T) = \tilde{A} + \tilde{B}\,T + \tilde{C}\,T^2,$$

which is independent of p, is modeled using a quadratic data fit. Higher order polynomial approximations are likewise possible, but quadratic fits are typically sufficient in the context of gas transport networks. The coefficients $\tilde{A}, \tilde{B}, \tilde{C}$ within the polynomial are called *heat capacity coefficients*. The real gas correction within the above formula is given by (see Doering, Schedwill, and Dehli (2012))

$$\Delta\tilde{c}_p(p,T) = -R\int_0^p \frac{1}{\tilde{p}}\left(2T\frac{\partial z}{\partial T} + T^2\frac{\partial^2 z}{\partial T^2}\right)\mathrm{d}\tilde{p}.$$

Here, R is again the universal gas constant, and $z = z(p,T)$ denotes the compressibility factor as introduced above.

A thermodynamical effect to mention at this point is the interdependence between changes in pressure p and changes in gas temperature T, which is known as the so-called *Joule–Thomson effect* (see, e.g., Oliveira (2013)). The Joule–Thomson effect is due to the interaction of gas molecules. When molecules are attracting each other and pressure is reduced, the distance of the molecules grows and mechanical work has to be performed to compensate for the attraction. The required energy comes from the kinetic energy of the molecules, which leads to a decrease of gas temperature. When the gas temperature exceeds the so-called *inversion temperature*, the molecules now repel each other, and an expansion of the gas leads to an increase of its temperature. However, in real-world gas transport networks these effects can usually be ignored and we can assume that the temperature increases as pressure is increased and vice versa. The change in the gas temperature caused by the Joule–Thomson effect can be obtained as an ODE solution involving the

Joule–Thomson coefficient μ_{JT}:

$$T_{\mathrm{out}} - T_{\mathrm{in}} = \int_{p_{\mathrm{in}}}^{p_{\mathrm{out}}} \mu_{\mathrm{JT}}(p,T)\,\mathrm{d}p, \tag{2.6}$$

with

$$\mu_{\mathrm{JT}}(p,T) = \frac{T^2}{p}\frac{R}{\tilde{c}_p}\frac{\partial z}{\partial T}. \tag{2.7}$$

The above formula can be derived by a straightforward application of the thermodynamical standard equation for real gases (2.3) to the definition of the Joule–Thomson coefficient (see, e.g., Oliveira (2013)).

Besides the physical properties introduced so far (especially pressure and temperature), the flow at every point in space and time of the gas in a network is a matter of particular interest. For operational purposes, gas flow is usually specified in terms of *mass flow* q (measured in kg/s). In contrast, network users, i.e., transport customers, usually nominate an amount of *thermal power* (measured in W). Thermal power P is related to mass flow via the equation

$$P = q H_{\mathrm{c}},$$

where H_{c} is the *calorific value*, which depends on the chemical composition of the gas. Thus, the properties of the gas mixtures supplied to the network are of significant importance. Usually, *low-calorific gas* (L-gas) with a calorific value of about 36 MJ/kg and *high-calorific gas* (H-gas) with a calorific value of about 41 MJ/kg are distinguished (see Table 1.3). Besides the calorific value, a few further important *gas quality parameters*, which have already been introduced, depend on the actual chemical composition of a gas mixture. These are the isobaric molar heat capacity coefficients $\tilde{A}$, $\tilde{B}$, $\tilde{C}$ (given in J mol^{-1}K$^{-\alpha}$, where $\alpha = 1,2,3$, respectively, correspond to $\tilde{A},\tilde{B},\tilde{C}$), the molar mass m (measured in kg/mol), as well as the pseudocritical pressure p_{c} and pseudocritical temperature T_{c} (measured in Pa and K, respectively).

There are junctions in the networks at which multiple pipes or other network elements join. At these junctions, we have *conservation of mass*:

$$\sum_{i\in I} q_i = \sum_{k\in K} q_k, \tag{2.8}$$

where I is the set of ingoing connections and K is the set of outgoing connections at the junction.

If heterogeneous gas mixtures are supplied to a nontrivial network, *gas mixtures* of different qualities meet at junctions: assume that gases with quality parameter vectors $X_i = (m_i, H_{\mathrm{c},i}, p_{\mathrm{c},i}, T_{\mathrm{c},i}, \tilde{A}_i, \tilde{B}_i, \tilde{C}_i)$ and mass flows q_i, $i \in I$, flow into a *junction* j. Then the mixed gas in the junction has parameters given by the vector

$$X_j = \frac{\sum_{i\in I} \frac{q_i}{m_i} X_i}{\sum_{i\in I} \frac{q_i}{m_i}}. \tag{2.9}$$

The gas temperature at junction j is determined by

$$T_j = \frac{\sum_{i\in I} c_{p,i} q_i T_i}{\sum_{i\in I} c_{p,i} q_i}. \tag{2.10}$$

Equation (2.10) can be derived from the *conservation of energy*; see Schmidt, Steinbach, and Willert (2014).

Figure 2.1. *A pipe with a diameter of* 1.2 m. *(Source: Open Grid Europe GmbH (OGE).)*

2.3 ▪ Gas network elements

In this section we describe the basic elements that commonly appear in gas networks, including virtual elements that we use as modeling devices. Every type of element has its own subsection in which we first give a phenomenological description of the element and then present a general mathematical formulation of the corresponding technical and physical effects. Since later chapters (Chapters 6–10) discuss concrete model formulations of the elements presented in this section, it serves as a foundation for the rest of the book.

2.3.1 ▪ Pipes

Natural gas transport networks consist mainly of pipelines (or pipes, for short); see Figure 2.1 for an illustration. Although the role of pipelines in gas networks is obvious, the impact of their design parameters on the gas flow needs closer consideration and will therefore be discussed in detail in the following. We first present a general pipe model and then give an approximate model for the stationary case.

2.3.1.1 ▪ The general case

From the technical point of view, the material and thickness of the pipe walls determine a limit for the pressure that the pipe is guaranteed to withstand. This limit is specified as *nominal pressure* of the pipe. Gas dynamics are influenced by the *length* L and the *diameter* D of a pipe. The longer the pipe is, the larger the pressure difference between its endpoints will be for a fixed amount of flow, and the larger the diameter is, the lower is the occurring pressure drop. In domestic gas transport networks, pipe diameters usually range between 10 cm and 1.4 m. A single gas network can contain large transport pipelines of more than 100 km of length and rather short pipes of lengths of less than 10 m.

The *inclination* of a pipe can have a significant impact on the gas flow. For example, in hilly areas, the endpoints of the pipe might be located at different levels causing a change of pressure due to gravity.

In general, the shape of the cross section of a pipe is also of importance. However, since virtually all gas pipelines in the considered transport networks are cylindrical, we restrict ourselves to this case and refer to Menon (2005) for a detailed description of the treatment of different pipe geometries. We describe selected properties of the pipe walls in the following.

Pipes are normally constructed of carbon steel with varying surface characteristics, depending on the manufacturing process. The pressure drop occurring in a pipe mainly arises because of friction, which is due to the roughness of the material of the inner pipe wall. The roughness of a pipe is a measure for the vertical deviation of the inner surface of the pipe wall from its ideal form and is thus measured in m. Pipes of different roughness also vary in their frictional resistance. Besides material roughness, the frictional resistance of a pipe is also influenced by other factors like, e.g., the presence of weld seams, the curvature of the pipe, corrosion processes, and the deposition of dirt and dust. All these factors are herein summarized in an "equivalent" surface roughness, the so-called *integral roughness* of the pipe. An estimate for the integral roughness of a real pipe can be obtained either experimentally or via measurements of optical or feeler instruments (see Kamnev (1966)).

Using the concept of the integral pipe roughness constitutes a simplification, in which effects, e.g., due to surface irregularities, are summarized. A detailed modeling of all such effects can be done for the investigation of gas dynamics in a single pipe using techniques from computational fluid dynamics. This leads to a system of three-dimensional PDEs with an additional choice of treating resistance due to the pipe walls; see, e.g., Anderson, Jr. et al. (2009), Batchelor (2000), and Landau and Lifshitz (1987) for an introduction and more details. In this book, however, we concentrate on simulating and optimizing entire gas networks, where many pipes and other elements are interconnected. Thus, a more macroscopic view is adequate in our context.

As mentioned earlier, we will restrict ourselves to the case of *cylindric* pipes and to the modeling of *one-dimensional* flow in the pipe direction x. Under these assumptions the mass flow q is related to gas density ρ and velocity v via

$$q = A \rho v, \tag{2.11}$$

where $A = D^2 \pi/4$ denotes the constant *cross-sectional area* of the pipe.

The gas dynamics within a single pipe is described by the following set of nonlinear, hyperbolic PDEs (see Feistauer (1993); Lurie (2008)), often referred to as *Euler equations*:

$$\frac{\partial \rho}{\partial t} + \frac{1}{A}\frac{\partial q}{\partial x} = 0, \tag{2.12}$$

$$\frac{1}{A}\frac{\partial q}{\partial t} + \frac{\partial p}{\partial x} + \frac{1}{A}\frac{\partial (q v)}{\partial x} + g \rho s + \lambda(q)\frac{|v|\, v}{2D}\rho = 0, \tag{2.13}$$

$$\begin{aligned} &A \rho c_p \left(\frac{\partial T}{\partial t} + v \frac{\partial T}{\partial x}\right) - A\left(1 + \frac{T}{z}\frac{\partial z}{\partial T}\right)\frac{\partial p}{\partial t} \\ &- A v \frac{T}{z}\frac{\partial z}{\partial T}\frac{\partial p}{\partial x} + A \rho v g s + \pi D c_{\mathrm{HT}} (T - T_{\mathrm{soil}}) = 0. \end{aligned} \tag{2.14}$$

The *continuity equation* (2.12) and the *momentum equation* (2.13) describe the *conservation of mass* and the *conservation of momentum*, respectively, while *energy conservation* is expressed by (2.14). Here, $v = v(x,t) \in \mathbb{R}$ is the velocity in the direction of the pipe, g denotes the *gravitational acceleration* (with standard value $9.806\,65\,\mathrm{m/s^2}$), and $s \in [-1,1]$ denotes the (constant) *slope* of the pipe, i.e., the tangent of its inclination angle. The *soil temperature* is denoted by T_{soil}, and c_{HT} denotes the *heat transfer coefficient*, given in $\mathrm{J/(m^2\,K\,s)}$, which expresses how heat is exchanged between the gas in the interior of the pipe and the surrounding of the pipe. This coefficient depends on the material and the thickness of the pipe wall. Thus, these have consequences on the fluid mechanical behavior of the gas.

Frictional forces are expressed via the so-called *friction factor* $\lambda = \lambda(q)$. For the computation of λ several phenomenological formulas are known from the literature. These formulas differ for *turbulent* and *laminar* flow, and hence depend on the *Reynolds number* (see Lurie (2008) and Saleh (2002)),

$$Re(q) = \frac{D}{A\eta}|q|, \tag{2.15}$$

where η denotes the *dynamic viscosity* of the gas. The flow is *turbulent* if $Re(q) \geq Re_{\text{crit}} \approx 2320$ and *laminar* otherwise.

In the laminar case, the formula of Hagen-Poisseuille (see Finnemore and Franzini (2002)) should be used to compute the friction factor:

$$\lambda(q) = \frac{64}{Re(q)}. \tag{2.16}$$

For the turbulent case, the implicit equation of *Prandtl and Colebrook* (also known as the Colebrook–White equation) constitutes the most accurate approximation of reality (Saleh (2002), Chap. 9):

$$\frac{1}{\sqrt{\lambda}} = -2\log_{10}\left(\frac{2.51}{Re(q)\sqrt{\lambda}} + \frac{k}{3.71D}\right). \tag{2.17}$$

A number of explicit approximations of (2.17) are known. We mention the equation of Hofer (see Hofer (1973); Mischner (2012)),

$$\lambda(q) = \left(-2\log_{10}\left(\frac{4.518}{Re(q)}\log_{10}\left(\frac{Re(q)}{7}\right) + \frac{k}{3.71D}\right)\right)^{-2}, \tag{2.18}$$

and the formula of Nikuradse (see Nikuradse (1933); Nikuradse (1950); Mischner (2012)):

$$\lambda = \left(2\log_{10}\left(\frac{D}{k}\right) + 1.138\right)^{-2}, \tag{2.19}$$

which can be derived from (2.18) for $Re \to \infty$.

In order to complete the system (2.11)–(2.14) of five unknowns and four equations, a suitable equation of state has to be added. As mentioned previously, we use the thermodynamical standard equation for real gases

$$p = \rho R_s T z \tag{2.20}$$

for this purpose. This is an equivalent reformulation of Eq. (2.3), which is obtained using the definition of *density*, $\rho = \tilde{n}\, m/V$, and the specific gas constant, $R_s = R/m$.

2.3.1.2 ▪ Approximations in the stationary and isothermal case

In this section we focus on the *stationary case*, i.e., the gas is in a steady state and all time derivatives in (2.12)–(2.14) are equal to zero. In this case, the continuity equation (2.12) simply states that the mass flow along the pipe is constant, i.e., $\partial_x q := \partial q/\partial x = 0$. Moreover, we only consider the *isothermal case* here. Thus, the gas temperature is considered to be constant, and the energy equation (2.14) can be neglected. The remaining momentum equation (2.13) states how the pressure change along the pipe depends on the amount of mass flow and the technical parameters of the pipe. In this book, we often use a further simplification of that equation: a quadratic approximation (see Bales (2005); Lurie (2008), which can be derived as follows.

We note first that the ram pressure term $\partial_x(qv)/A$ contributes less than one percent to the sum of all terms under normal operating conditions; see Wilkinson et al. (1964). Hence we assume that $\partial_x(qv)/A$ can be neglected. Moreover, we assume that the gas temperature T and the compressibility factor z can be approximated by suitable constants along the entire pipe, say, by mean values T_m and z_m. Finally, assume that only pipes with constant slope s are considered. Then the stationary momentum equation (2.13) can be rewritten as

$$\frac{\partial p}{\partial x} + g\,\rho\, s + \lambda(q)\frac{|v|\,v}{2D}\rho = 0. \tag{2.21}$$

Lemma 2.1. *For $s \neq 0$, the solution $p(x)$ to (2.21) with initial value $p(0) = p_\mathrm{in}$ is given by*

$$p(x)^2 = \left(p_\mathrm{in}^2 - \tilde{\Lambda}\,|q|\,q\,\frac{e^{\tilde{S}x}-1}{\tilde{S}}\right)e^{-\tilde{S}x} \tag{2.22}$$

with

$$\tilde{S} := \frac{2\,g\,s}{R_\mathrm{s}\,z_\mathrm{m}\,T_\mathrm{m}}, \quad \tilde{\Lambda} := \lambda(q)\frac{R_\mathrm{s}\,z_\mathrm{m}\,T_\mathrm{m}}{A^2 D}.$$

Proof. In (2.21), we replace the gas velocity v by the mass flow q using (2.11) and the gas density ρ by the pressure p using the equation of state (2.20). This yields

$$\frac{\partial p}{\partial x} + g\,\frac{p}{R_\mathrm{s}\,z_\mathrm{m}\,T_\mathrm{m}}s + \lambda(q)\,\frac{|q|\,q}{2A^2 D}\,\frac{R_\mathrm{s}\,z_\mathrm{m}\,T_\mathrm{m}}{p} = 0,$$

where we use the assumption that the gas temperature and compressibility factor are constants T_m and z_m, respectively. Multiplication by $2\,p$ leads to

$$\frac{\partial}{\partial x}p^2 + \tilde{S}\,p^2 = -\tilde{\Lambda}\,|q|\,q.$$

If we now substitute $y = p^2$, we end up with the first-order linear ordinary differential equation (ODE)

$$\frac{\partial}{\partial x}y + \tilde{S}\,y = -\tilde{\Lambda}\,|q|\,q, \quad y(0) = p_\mathrm{in}^2. \tag{2.23}$$

This ODE can be solved analytically by "variation of constants," and we arrive at

$$y(x) = p(x)^2 = \left(-\tilde{\Lambda}\,|q|\,q\,\frac{1}{\tilde{S}}e^{\tilde{S}x} + p_\mathrm{in}^2 + \tilde{\Lambda}\,|q|\,q\,\frac{1}{\tilde{S}}\right)e^{-\tilde{S}x},$$

where the last two terms in parentheses represent the integration constant obtained from the initial value $y(0) = p_\mathrm{in}^2$. This concludes the proof. ☐

By evaluating the solution of Eq. (2.22) at $x = L$ (with $p(L) = p_\mathrm{out}$) and fixing the notation $\Lambda := \tilde{\Lambda}L$ and $S := \tilde{S}L$, we finally obtain a well-known relationship of inlet and outlet pressures and the mass flow through the pipe (see, e.g., Lurie (2008)),

$$p_\mathrm{out}^2 = \left(p_\mathrm{in}^2 - \Lambda\,|q|\,q\,\frac{e^S-1}{S}\right)e^{-S} \tag{2.24}$$

with

$$\Lambda = \lambda(q)\frac{R_s z_m T_m L}{A^2 D} = \left(\frac{4}{\pi}\right)^2 \lambda(q)\frac{R_s z_m T_m L}{D^5}, \quad S = \frac{2 g s L}{R_s z_m T_m}. \tag{2.25}$$

Note that the pressure $p(x)$ according to Eq. (2.22), and hence $p(L) = p_{out}$ according to Eq. (2.24), are not defined for horizontal pipes, i.e., if the slope s is zero. The solution for this case is obtained by solving the (trivial) ODE (2.23) with $\tilde{S} = 0$, or by taking the limit for $s \to 0$ (equivalently $\tilde{S} \to 0$) in (2.22) using l'Hôpital's rule.

Lemma 2.2. *For $s = 0$, the solution $p(x)$ to Eq.* (2.21) *with initial value $p(0) = p_{in}$ is given by*

$$p(x)^2 = p_{in}^2 - x\,\tilde{\Lambda}\,|q|\,q, \tag{2.26}$$

with $\tilde{\Lambda}$ as defined in Lemma 2.1.

Evaluating Eq. (2.26) at $x = L$ now yields the pressure loss formula for horizontal pipes:

$$p_{out}^2 = p_{in}^2 - \Lambda\,|q|\,q. \tag{2.27}$$

It remains to choose appropriate approximations for the constant values z_m and T_m as required by Lemma 2.1. A good estimate of the mean values would be most suitable. Since $z_m = z(p_m, T_m)$ is typically defined by Eq. (2.4) or Eq. (2.5), z_m can be obtained from an adequate mean value of p_m. In fact, there exists an elegant closed-form expression for p_m which is often used throughout the literature (see, e.g., Saleh (2002)). It depends only on p_{in} and p_{out} and is superior to a simple arithmetic mean.

Lemma 2.3. *Let $p(x)$ be given as in Lemma* 2.2, *and let*

$$p_m := \frac{1}{L}\int_0^L p(x)\,\mathrm{d}x$$

be the mean pressure along the pipe. Then

$$p_m = \frac{2}{3}\left(p_{in} + p_{out} - \frac{p_{in}\,p_{out}}{p_{in} + p_{out}}\right). \tag{2.28}$$

Proof. Initially, we seek a closed form expression for $p(x)$ independent of q and any mean values. To obtain such a formula we eliminate the flow term by multiplying Eq. (2.26) by L and subtracting Eq. (2.27) multiplied by x. After solving for $p(x)$, we can rewrite the mean pressure as

$$p_m = \frac{1}{L}\int_0^L p(x)\,\mathrm{d}x = \frac{1}{L}\int_0^L \sqrt{p_{in}^2 - \frac{x}{L}(p_{in}^2 - p_{out}^2)}\,\mathrm{d}x.$$

Evaluating the integral then yields the desired formula (2.28). ◻

Without involving the energy equation (or approximative solutions of it), a simple arithmetic mean for the average temperature

$$T_m := \frac{1}{2}(T_{in} + T_{out}) \tag{2.29}$$

is the most adequate choice.

Finally, we would like to mention that Eq. (2.24) (or (2.27)) may be equivalently formulated in terms of the *volumetric flow rate under normal conditions* Q_0, using the relation

$$q = A\rho v = \rho_0 Q_0.$$

Here, ρ_0 denotes the normal density, which is the density under *normal conditions*. Both can be obtained from the equation of state for real gases (2.20), and hence ρ_0 is simply a constant, at least if mixing effects are ignored.

2.3.1.3 ▪ Further effects

We remark that, as it is the case with virtually any mathematical model for complex physical phenomena, not all practically relevant effects can be taken into account. For example, unexpected pressure losses can be caused by leakages, which mostly occur due to corrosion. To check for leaks, to clean the pipes, and for measuring purposes, gas transport companies make use of so-called *"pipeline pigs."* These are sets of tools that are installed within the pipelines. For further details on this topic, see, e.g., Kennedy (1993).

2.3.2 ▪ Resistors

In addition to the pressure loss resulting from friction of the flow through the pipes, there are certain gas properties and network components that also induce a pressure loss, which has to be accounted for. Causes for such pressure losses are, e.g., flow diversion and turbulence in shaped pieces, measurement devices, curvature of the piping within compressor stations and pressure regulators, filter systems, reduced radii, and partially closed valves. These effects are often quite complicated, and no accurate models are available for most of them. Resistors are a surrogate modeling tool used for representing these forms of pressure loss.

Resistors influence pressure in the same way for both directions of the flow. Thus, p_{in} and p_{out} in the following formulas refer to the pressure at the inlet and the outlet node of the resistor, which depend of the flow direction. Similarly, v_{in} refers to the gas velocity at the inlet.

There are two forms of resistors being used. In the first form a resistor causes a nonlinear pressure loss according to a type of Darcy–Weisbach formula (Finnemore and Franzini (2002); Lurie (2008)) with parameters ζ and D. The pressure loss depends on the drag factor ζ, the density of the entering gas ρ_{in}, and its velocity v_{in}:

$$p_{\text{in}} - p_{\text{out}} = \frac{1}{2}\zeta\, \rho_{\text{in}}\, v_{\text{in}}^2, \tag{2.30}$$

where the velocity is computed from the mass flow q and the resistor's fictitious cross-sectional area, $A = D^2\,\pi/4$, as follows:

$$v_{\text{in}} = \frac{q}{A\rho_{\text{in}}}. \tag{2.31}$$

The corresponding parameters have to be fitted to measurements of the actual pressure loss. This Darcy–Weisbach form can be used if there are sufficient measurement data for the affected region. Otherwise, a simpler form of resistor model is used, based on an estimate of the pressure loss.

In this second form, resistors incur a fixed pressure loss ξ in the flow direction:

$$p_{\text{in}} - p_{\text{out}} = \operatorname{sgn}(q)\xi, \tag{2.32}$$

Figure 2.2. *A ball valve. (Source: OGE.)*

where

$$\operatorname{sgn}(x) = \begin{cases} 1 & \text{if } x > 0, \\ 0 & \text{if } x = 0, \\ -1 & \text{if } x < 0. \end{cases}$$

Due to the Joule–Thomson effect (see Section 2.2), the temperature of the gas decreases when passing through a resistor.

2.3.3 ▪ Valves

Valves are active network elements that can be controlled by the network operators. From our point of view, they can be closed or open. In practice, valves can also be *partially closed* in order to control the gas velocity. In this case, we model the partly closed valve as a resistor (see Section 2.3.2).

Valves are used to route the gas flow for parts of the transport network or to block gas flow for maintenance in subnetworks. In addition, they are frequently used in compressor or control valve stations for inner station piping (see Section 2.4.1 and Section 2.4.3). Depending on the concrete valve type, the switching between these states is technically realized in different ways. Most of the valves in the transport networks under consideration are *gate* or *ball valves*. The opening and closing of gate valves is simply realized by raising or lowering the gate wall. If the gate wall is raised, the gas flows through the element, while a closed valve blocks the complete gas flow. Ball valves control the gas flow with a ball that has a centered cylindrical hole; see Figure 2.2 for an illustration. If the hole of the ball is in line with the ends of the valve, the gas flows through the element. If the hole is rotated by 90°, the valve blocks the gas flow. In practice, only balls with restricted diameters are assembled. Because the used valves have to fit the ambient pipes, this maximum diameter is an upper bound for pipe diameters, too.

For the following, consider a valve with mass flow q and respective inlet and outlet pressures p_{in} and p_{out}. Open valves lead to identical values of the gas state quantities pressure p, temperature T, and density ρ:

$$p_{\text{in}} = p_{\text{out}}, \tag{2.33}$$

$$\rho_{\text{in}} = \rho_{\text{out}}, \tag{2.34}$$

$$T_{\text{in}} = T_{\text{out}}. \tag{2.35}$$

Because valves are elements with negligible length, there is only insignificant friction that we neglect in (2.33)–(2.35).

Closed valves prevent gas flow, yielding decoupled gas states at both sides of the valve:

$$q = 0, \quad (p_{\text{in}}, T_{\text{in}}), (p_{\text{out}}, T_{\text{out}}) \text{ decoupled.} \tag{2.36}$$

2.3.4 ▪ Control valves

Larger transport pipes are usually operated at higher pressure than pipes in the distribution parts of the network, which have a smaller diameter and smaller nominal pressure. In order to interconnect network parts operated at higher pressure with those operated at lower pressure and also to add a means of control of the flow, *control valves*, also known as *pressure regulators*, are used. There are two types of control valves: with remote access (automated) and without remote access (nonautomated).

Control valves can be *closed*, so that there is no flow and the inlet and outlet pressures are decoupled. If the control valve is *active*, the pressure at its outlet can be reduced to a given controllable value. In this case, control valves have a fixed working direction.

The pressure reduction is accomplished by a variable valve, being capable of restricting the flow. Its degree of opening, and thus the amount of flow through it, is usually controlled by a diaphragm actuator in combination with a compression spring. The outlet pressure can be controlled by changing the compression of the spring via a screw, thus adjusting the force that the spring exerts to the diaphragm actuator. For automated control valves, the change of the force of the spring can be remotely controlled by the dispatcher by an engine that is attached to the handle of the spring. The active/closed state and the controlled output pressure for control valves with remote access thus can be freely adjusted to the load requirements of the network. In network operation planning, the outlet pressure can be considered a control variable in this case.

For nonautomated control valves (a remote access device is not present) only a manual change is possible, requiring a person to be sent to the control valve. While the downstream pressure of control valves with remote access can be controlled directly, control valves without remote access are designed to reduce the downstream pressure to a preset pressure. In this case, the preset pressure of the control valve has to be considered fixed for short-term planning purposes. This is accomplished by the device under the conditions that the upstream pressure is at least as high as the preset value and that the downstream pressure is not greater than this value. If the downstream pressure rises above this threshold, the control valve without remote access closes automatically. If the downstream pressure is less than or equal to the threshold and the upstream pressure drops below this threshold value, the control valve without remote access opens fully and is *in bypass*. In bypass, the pressure of the gas is not affected and a flow in the opposite direction is possible. It is possible to have a bypass state for automated control valves as well; this is usually modeled on the level of control valve stations; see Section 2.4.1.

For a more detailed description of the construction of control valves and the corresponding legal requirements see Cerbe (2008).

For both types of control valves, the gas temperature drops as a consequence of the Joule–Thomson effect (see Section 2.2) due to the expansion of the gas when passing through a control valve. If the pressure change and thus the temperature drop is substantial, gas hydrates might fall out or the instrument could even freeze. For this reason, a control valve in most cases is combined with a gas *preheater*; see Section 2.4.1. Preheaters use the gas, e.g., by catalytic chemical reactions, to increase the temperature of the gas before entering the variable valve. If such a device is present, the outlet temperature of the control valve is determined by the operation of the preheater. Otherwise, the temperature change follows Eq. (2.6).

Control valves may incur a restriction on the flow rate if they are combined with measurement devices. In order to express the pressure loss due to filtering and inner-station piping, up- and downstream resistors (see Section 2.3.2) may be used to model these effects; see Section 2.4.1.

2.3.4.1 ▪ Control valves with remote access

The different states of control valves with remote access have the following consequences. Let q be the flow through a control valve and p_{in} and p_{out}, respectively, the inlet and outlet pressures. If the control valve is *closed*, flow is blocked and pressures are decoupled:

$$q = 0, \quad p_{\text{in}},\ p_{\text{out}} \text{ decoupled.}$$

If the control valve is *active*, it is capable of reducing the pressure by an amount within the range $[\underline{\Delta}, \overline{\Delta}]$, leading to the following model:

$$0 \le \underline{\Delta} \le p_{\text{in}} - p_{\text{out}} \le \overline{\Delta}, \quad q \ge 0. \tag{2.37}$$

2.3.4.2 ▪ Control valves without remote access

For a control valve without remote access and a preset downstream pressure p_a^{set}, the following relations hold:

$$\begin{array}{rcll l}
p_{\text{out}} > p_a^{\text{set}} & \Longrightarrow & q = 0, & p_{\text{in}} \text{ arbitrary}, & (\textit{closed}) \\
p_{\text{out}} > p_{\text{in}} & \Longrightarrow & q = 0, & & (\textit{closed}) \\
p_{\text{in}} > p_a^{\text{set}} \text{ and } p_{\text{out}} \le p_a^{\text{set}} & \Longrightarrow & q \ge 0, & p_{\text{out}} = p_a^{\text{set}}, & (\textit{active}) \\
p_{\text{out}} \le p_{\text{in}} \le p_a^{\text{set}} & \Longrightarrow & q \text{ arbitrary}, & p_{\text{out}} = p_{\text{in}}. & (\textit{bypass})
\end{array}$$

2.3.5 ▪ Compressor machines and drives

Compressor machines are among the most important and complex elements in gas transport networks. They are used to increase the pressure of the incoming gas to a higher outflow pressure. Thus, compressor machines satisfy the need to overcome pressure loss caused by friction in pipes (see Section 2.3.1) and to transport gas over long distances. We sometimes also simply use the term compressor for a compressor machine. Every compressor machine has an associated drive (see Figure 2.3). It is possible that more than one compressor is powered by the same drive. In present-day gas transport networks, one mainly finds *turbo compressors* and *piston compressors* in combination with one of four drive types:

- ▹ gas turbines,
- ▹ gas driven motors,

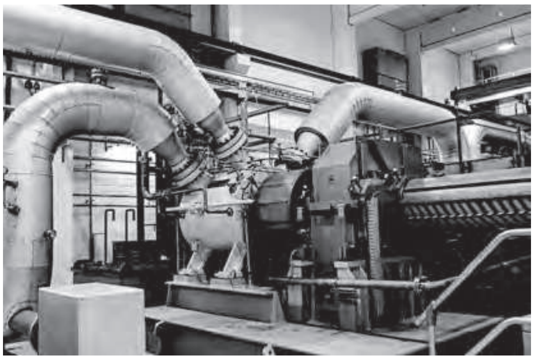


Figure 2.3. *A compressor machine. (Source: Schmidt, Steinbach, and Willert (2014).)*

▹ electric motors, and
▹ steam turbines.

These will be described in Section 2.3.5.4.

2.3.5.1 ▪ Compressor machines

Compressor machines admit certain feasible combinations of *throughput* (measured in volumetric flow Q as defined below by Eq. (2.38)) and *specific change in adiabatic enthalpy* H_{ad} (as derived below by Eq. (2.42)). The set of all possible combinations of throughput and specific change in adiabatic enthalpy is called the *feasible operating range* of the machine. The volumetric flow depends on the mass flow q through the machine and the inflow gas density ρ_{in}:

$$q = Q\rho_{\mathrm{in}}. \tag{2.38}$$

In order to derive the specific change in adiabatic enthalpy, we have to discuss the physical process of compression by a compressor machine. To this end, we need some fundamental thermodynamical quantities. Consider a certain quantity of gas that undergoes some thermodynamical process. The *enthalpy* $\tilde{H}$ of such a physical system is a measure of its total energy, composed of *internal energy* U and the *work* $W = p\,V$. In differential form, enthalpy is defined by

$$\mathrm{d}\tilde{H} = \mathrm{d}U + \mathrm{d}(p\,V) = \mathrm{d}U + V\,\mathrm{d}p + p\,\mathrm{d}V;$$

see, e.g., Tahir-Kheli (2012). By the first law of thermodynamics (see again Tahir-Kheli (2012)) the change in internal energy during the process, ignoring chemical reactions, is

$$\mathrm{d}U = \mathrm{d}\tilde{Q} - p\,\mathrm{d}V,$$

where $\tilde{Q}$ denotes the *heat exchange*, i.e., energy transferred between the system and its surroundings. Thus, the change in enthalpy can be written as

$$\mathrm{d}\tilde{H} = \mathrm{d}\tilde{Q} + V\,\mathrm{d}p.$$

If the compression process is done thermally isolated from the surrounding, no external heat exchange can occur. Such a process with $\mathrm{d}\tilde{Q} = 0$ is called *adiabatic*. Under the assumptions of an adiabatic compression process, the change in enthalpy reduces to

$$\mathrm{d}\tilde{H}_{\mathrm{ad}} = V\,\mathrm{d}p,$$

where the subscript "ad" indicates an adiabatic compression process. Consequently, the change in enthalpy for an adiabatic compression process with inlet pressure p_{in} and outlet pressure p_{out} is

$$\tilde{H}_{\text{ad}} = \int_{p_{\text{in}}}^{p_{\text{out}}} V \, \mathrm{d}p. \tag{2.39}$$

According to Eq. (2.39), the change in adiabatic enthalpy that is required to increase the inlet pressure p_{in} of the gas to an outlet pressure p_{out} is equivalent to the work done on the system for this change in pressure.

In the following, let V_{in} and V_{out} denote the inlet and outlet volumes of the quantity of gas under consideration. Since the compression is adiabatic and reversible according to the second law of thermodynamics, it is *isentropic*. For an isentropic compression process one can derive the relationship

$$p_{\text{in}} V_{\text{in}}^{\varkappa} = p_{\text{out}} V_{\text{out}}^{\varkappa}, \tag{2.40}$$

using again the first and second laws of thermodynamics and the assumption of an ideal gas Tahir-Kheli (2012). The constant $\varkappa \neq 0$ is called *isentropic exponent*; a suitable choice for real gases will be discussed below. Solving Eq. (2.40) for V_{out} yields

$$V_{\text{out}} = \left(\frac{p_{\text{in}}}{p_{\text{out}}}\right)^{\frac{1}{\varkappa}} V_{\text{in}}. \tag{2.41}$$

Since this equation is valid for arbitrary p_{out} and V_{out}, we can apply Eq. (2.41) with $p_{\text{out}} = p$ and $V_{\text{out}} = V$ to Eq. (2.39):

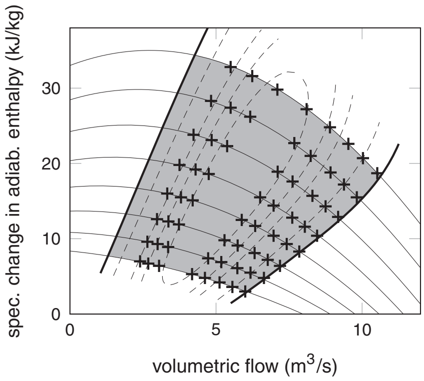
$$\begin{aligned}
\tilde{H}_{\text{ad}} &= \int_{p_{\text{in}}}^{p_{\text{out}}} \left(\frac{p_{\text{in}}}{p}\right)^{\frac{1}{\varkappa}} V_{\text{in}} \, \mathrm{d}p \\
&= p_{\text{in}}^{\frac{1}{\varkappa}} V_{\text{in}} \int_{p_{\text{in}}}^{p_{\text{out}}} \frac{1}{p^{\frac{1}{\varkappa}}} \, \mathrm{d}p \\
&= p_{\text{in}}^{\frac{1}{\varkappa}} V_{\text{in}} \frac{\varkappa}{\varkappa - 1} \left[p^{\frac{\varkappa-1}{\varkappa}} \right]_{p_{\text{in}}}^{p_{\text{out}}} \\
&= p_{\text{in}} V_{\text{in}} \frac{\varkappa}{\varkappa - 1} \left[\left(\frac{p_{\text{out}}}{p_{\text{in}}}\right)^{\frac{\varkappa-1}{\varkappa}} - 1 \right].
\end{aligned}$$

Now we can apply the thermodynamical standard equation for real gases (2.3) to eliminate $p_{\text{in}} V_{\text{in}}$ and obtain

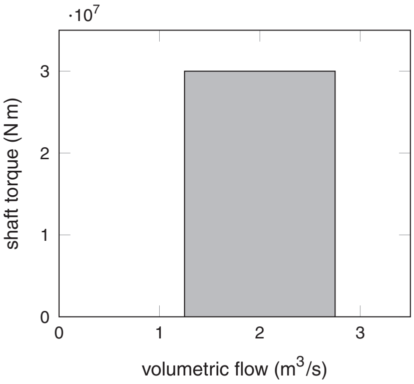
$$\tilde{H}_{\text{ad}} = \tilde{n} R T_{\text{in}} z_{\text{in}} \frac{\varkappa}{\varkappa - 1} \left[\left(\frac{p_{\text{out}}}{p_{\text{in}}}\right)^{\frac{\varkappa-1}{\varkappa}} - 1 \right].$$

Here, the compressibility factor is denoted by z_{in}, and T_{in} is the gas temperature at the inlet node of the compressor.

The (adiabatic) enthalpy $\tilde{H}$ ($\tilde{H}_{\text{ad}}$) as derived above is proportional to the size of the thermodynamical system; more specifically, to the amount of substance $\tilde{n}$. It is convenient to introduce a *specific enthalpy* $H := \tilde{H}/M$, which is independent of the amount of substance $\tilde{n}$ and independent of the mass M. Finally, with the relation $M = \tilde{n} m$, one



(a) Characteristic diagram of a turbo compressor. Specific change in adiabatic enthalpy H_{ad} vs. volumetric flow rate Q: Dashed lines represent isolines for adiabatic efficiency η_{ad}, thin solid lines represent isolines for compressor speed n. The left thick solid line represents the surgeline, the right thick solid line represents the chokeline. All curves are the result of least-squares fits with respect to the measurements "+".



(b) Characteristic diagram of a piston compressor.

Figure 2.4. *Characteristic diagrams of compressors (feasible operating ranges are marked gray). (Source: OGE.)*

obtains the fundamental formula for the specific change in adiabatic enthalpy of a compression process:

$$H_{\mathrm{ad}} := \frac{\tilde{H}_{\mathrm{ad}}}{M} = R_{\mathrm{s}}\, T_{\mathrm{in}}\, z_{\mathrm{in}} \frac{\varkappa}{\varkappa - 1} \left[\left(\frac{p_{\mathrm{out}}}{p_{\mathrm{in}}} \right)^{\frac{\varkappa-1}{\varkappa}} - 1 \right], \tag{2.42}$$

where $R_{\mathrm{s}} = R/m$ is the specific gas constant as introduced previously. The derived unit of specific change in adiabatic enthalpy H_{ad} is J/kg. Note that the *adiabatic head* (measured in m), frequently also abbreviated as H_{ad} in the literature, is H_{ad}/g.

The isentropic exponent $\varkappa$ used above in fact depends on gas pressure and temperature. Several approximations of $\varkappa$ exist that differ in complexity and accuracy; see Section 10.1.10.2 and Schmidt, Steinbach, and Willert (2014) for a more detailed discussion. Except for Chapter 10, we will approximate the isentropic exponent by the constant 1.296 in this book.

To realize different combinations of throughput and specific change of adiabatic enthalpy within the feasible operating range, compressors can be operated at different speeds n influencing both specific change in adiabatic enthalpy and throughput. It is common in gas engineering to visualize feasible operating ranges in so-called *characteristic diagrams* (see, e.g., Odom and Muster (2009); Percell and Ryan (1987). Depending on the machine type (e.g., turbo compressor or piston compressor), the characteristic diagram may also depend on the *adiabatic efficiency* η_{ad} of the machine, i.e., the quotient of conducted and emitted power, as we explain next; see Figure 2.4 for examples.

The *power P* required for compression depends on the amount of compressed gas (mass flow q), the realized specific change in adiabatic enthalpy H_{ad}, and the *adiabatic* *efficiency* $\eta_{\text{ad}} \in [0,1]$ of the compression process:

$$P = \frac{q\,H_{\text{ad}}}{\eta_{\text{ad}}}. \tag{2.43}$$

Here, $q\,H_{\text{ad}}$ is the theoretical power required by the adiabatic compression process, and η_{ad} is an estimate used to describe the deviation from the actual power P (e.g., according to mechanical losses) for a real machine.

Together with the specific *energy consumption rate* b of the corresponding drive (see below), the power required for compression determines the amount of energy that is consumed by the machine. Depending on the corresponding drive, the consumed power is either electricity or gas from the network used as fuel. Electric energy is delivered by specific electric motors, whereas gas is transformed to mechanical energy by gas turbines or gas driven motors (see Section 2.3.5.4 for a detailed description of drives). The amount of electric energy is directly given by the specific energy consumption rate b. The amount of fuel gas consumption is given by

$$q_{\text{fuel}} = \frac{b\,m}{H_{\text{u}}}, \tag{2.44}$$

where H_{u} is the *lower calorific value* of the gas (see Cerbe (2008)). As before, m denotes the molar mass of the gas.

The compression of the gas leads to an increase of the gas temperature. To prevent overheating, most of the compressor groups (see Section 2.4.2) contain a *gas cooler* that decreases the outflow temperature if a threshold is exceeded. This threshold mostly depends on the heat resistance of the internal coating of the pipes. Standard thresholds are about 35 °C to 50 °C. Because the gas temperature increase depends on the chosen operation mode of the group, the heat resistance may exclude some modes from the principally possible ones.

At turbo and piston compressors, two different technical processes increase the gas pressure. Turbo compressors add energy to the gas by a rotating edge runner. Here, the conducted energy depends on the rotational speed of the runner. In contrast to that, the gas gets compressed by a crankshaft in piston compressors. Turbo compressors are capable of compressing larger amounts of gas, while piston compressors achieve higher compression ratios for smaller amounts of gas. Since piston compressors can be operated for very long time periods, they are typically used as storage compressors. Typical values of the pressure ratio realized by turbo compressors vary between 1.35 to 1.5, whereas piston compressors can achieve values up to 4. Typical maximum power values may vary between 5 MW and 25 MW.

Finally, we remark that it is possible to operate a compressor beyond the left boundary of its characteristic diagram. This is called *pump prevention*, since it is realized by the reinsertion of compressed gas in order to keep a certain level of flow through the compressor. For this exceptional mode of operation, gas coolers are essential to prevent overheating.

2.3.5.2 ▪ Turbo compressors

An exemplary characteristic diagram of a turbo compressor can be seen in Figure 2.4(a). We follow the standard technique to model all curves in the characteristic diagram of a turbo compressor by quadratic polynomials (see Odom and Muster (2009)),

$$\psi(x;a) = a_0 + a_1 x + a_2 x^2, \tag{2.45}$$

or biquadratic polynomials,

$$\chi(x,y;A) = \begin{pmatrix} 1 \\ x \\ x^2 \end{pmatrix}^T \begin{pmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \\ a_{20} & a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} 1 \\ y \\ y^2 \end{pmatrix}. \tag{2.46}$$

The coefficients $a \in \mathbb{R}^3$ and $A \in \mathbb{R}^{3\times 3}$ are obtained from least-squares–based data fits for given technical measurements of the compressor machine. In the following, a superscript term on a or A specifies the coefficient vector or matrix of the quantity to which it corresponds. The *isolines* of compressor speed $n \in [\underline{n}, \overline{n}]$ are given by the implicit equation

$$H_{\text{ad}} = \chi(Q, n; A^{\text{speed}}). \tag{2.47}$$

They determine the lower boundary (with minimum compressor speed $\underline{n}$) and upper boundary (with maximum compressor speed $\overline{n}$) in Figure 2.4(a). The isolines of adiabatic efficiency,

$$\eta_{\text{ad}} = \chi(Q, n; A^{\text{eff}}), \tag{2.48}$$

determine the power required to realize a given working point of the compressor (see (2.43)). The left and right boundaries of the characteristic diagram are given by the *surgeline* and *chokeline*, respectively,

$$\psi(Q; a^{\text{surge}}) \geq H_{\text{ad}}, \quad \psi(Q; a^{\text{choke}}) \leq H_{\text{ad}}. \tag{2.49}$$

2.3.5.3 ▪ Piston compressors

Piston compressors are characterized by box-shaped feasible operating ranges in the coordinates volumetric flow rate Q and *shaft torque* M; see Figure 2.4(b) for an illustration. The shaft torque is defined as

$$M := \frac{V_{\text{o}} H_{\text{ad}}}{2\pi \eta_{\text{ad}}} \rho_{\text{in}}. \tag{2.50}$$

Here, V_{o} is the operating volume of the piston compressor. The volumetric flow rate is proportional to the speed of the machine,

$$Q = V_{\text{o}} n, \quad n \in [\underline{n}, \overline{n}].$$

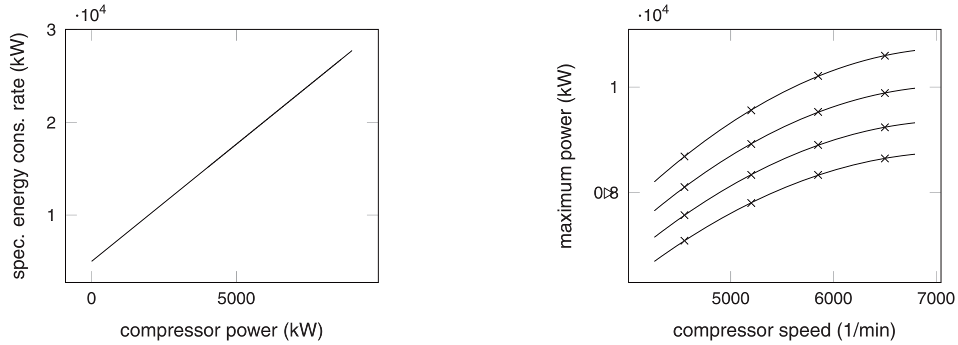
Depending on the technical specification of the piston compressor, the *maximum torque* is (implicitly) given by one of the following three restrictions:

$$\frac{p_{\text{out}}}{p_{\text{in}}} \leq \overline{\varepsilon}, \quad p_{\text{out}} - p_{\text{in}} \leq \overline{\Delta p}, \quad \text{or} \quad M \leq \overline{M}.$$

Here, $\overline{\varepsilon}$ and $\overline{\Delta p}$ are the maximal pressure ratio and the maximal pressure increase, respectively. Note that p_{in} and p_{out} are connected to M via H_{ad} (see (2.50)).

2.3.5.4 ▪ Drives

Drives deliver the energy required for the compression realized by compressor machines. We distinguish drives that produce energy from electricity or from gas. If the energy is produced from gas, the required amount of gas depends on the gas state and its calorific value (see (2.44)). The gas is taken from the inlet of the corresponding compressor and, thus, has the pressure of the incoming flow.



(a) Specific energy consumption rate b vs. the compressor power P.

(b) Maximum power $\overline{P}$ vs. compressor speed n for different ambient temperatures T_{amb}.

Figure 2.5. *Characteristic diagrams of a gas turbine. (Source: Schmidt, Steinbach, and Willert (2014).)*

As for compressors, we follow the common approach in industry and model all physical and technical relationships with quadratic or biquadratic functions (see LIWACOM (2004)). Every drive has a specific *energy consumption rate*

$$b(P) = \psi(P; a^{\text{energy}}) \tag{2.51}$$

and a *maximum power* $\overline{P}$ that can be delivered for the compression process. Depending on the concrete drive type, the latter may be a function of compressor speed and the ambient temperature T_{amb} at the compressor,

$$\overline{P}(n, T_{\text{amb}}) = \chi(n, T_{\text{amb}}; A^{\text{max-power}}), \tag{2.52}$$

or independent of the ambient temperature, i.e.,

$$\overline{P}(n) = \psi(n; a^{\text{max-power}}). \tag{2.53}$$

This value $\overline{P}$ restricts the power available for compression,

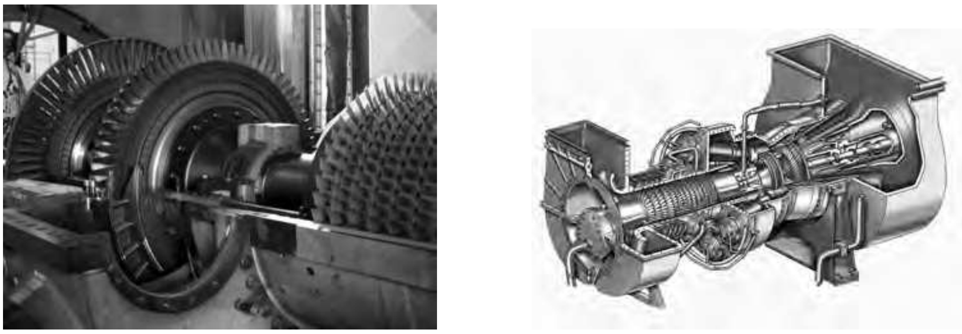
$$P \leq \overline{P}.$$

As for characteristic diagrams of turbo compressors, the coefficients in (2.51), (2.52), and (2.53) are obtained from least-squares data fits.

Figure 2.5 shows typical plots of the specific energy consumption rate function b and the maximum power function $\overline{P}$ for a gas turbine. Here, the maximum power function depends on the ambient temperature T_{amb}, too, leading to different maximum power functions for different values of T_{amb}. Figure 2.6 shows an example for a gas turbine as used in drives.

2.4 ▪ Gas network structures

In real-world gas transport networks, the basic network elements described in the last section are combined in complex ways to build a flexible network structure that may support various transport situations. It is thus appropriate to introduce further modeling elements that allow us to express the high-level structure of groups of basic network elements. These high-level modeling elements are discussed in this section. This includes



(a) Example of a gas turbine. (Source: OGE.)

(b) Schematic plot of a gas turbine. (Source: Siemens AG.) © Siemens AG

Figure 2.6. *Gas turbines.*

Table 2.1. *Technical symbols of gas network elements.*

Network element	Symbol
Resistor	
Valve	
Control valve	
Compressor (group)	

control valve stations (Section 2.4.1), compressor groups (Section 2.4.2), and compressor stations (Section 2.4.3).

Other common substructures are *loops*, which are parallel pipes. They are usually built with connecting valves at regular distances, enabling different ways of operating them. They may be used as two parallel pipes at the same conditions (i.e., pressure levels) by opening all the valves, or independently at different conditions by closing all the valves. By closing a subset of the valves, it is even possible to use parts of a loop in both directions. In contrast to compressor stations, we do not have a high-level model for loops. Instead, we model them directly by using the corresponding network structure made up of pipes and valves.

For a graphical representation of the network elements in the following, we use the symbols in Table 2.1.

2.4.1 ▪ Control valve stations

As mentioned in Section 2.3.4, control valves are usually combined with preheaters. Moreover, valves might be used to control flow and allow a bypass of the control valve. To represent elaborate piping in connection with the control valve, resistors can be added as well. This yields a subnetwork, which we call a *control valve station*; see Figure 2.7 for an example.

Note that the concrete layout may depend on the underlying mathematical optimization model and network design. For instance, Figure 2.7 does not explicitly contain a preheater, in contrast to the modeling of Chapter 10 in Figure 10.4; the latter, however, does not contain an explicit bypass. Consequently, control valve stations represent an

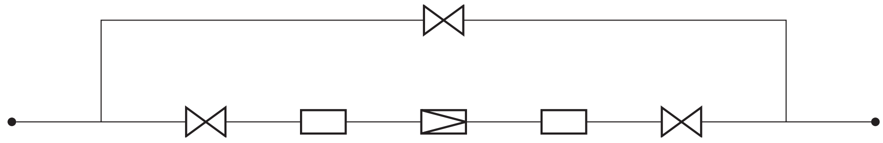


Figure 2.7. *Diagram of a control valve station.*

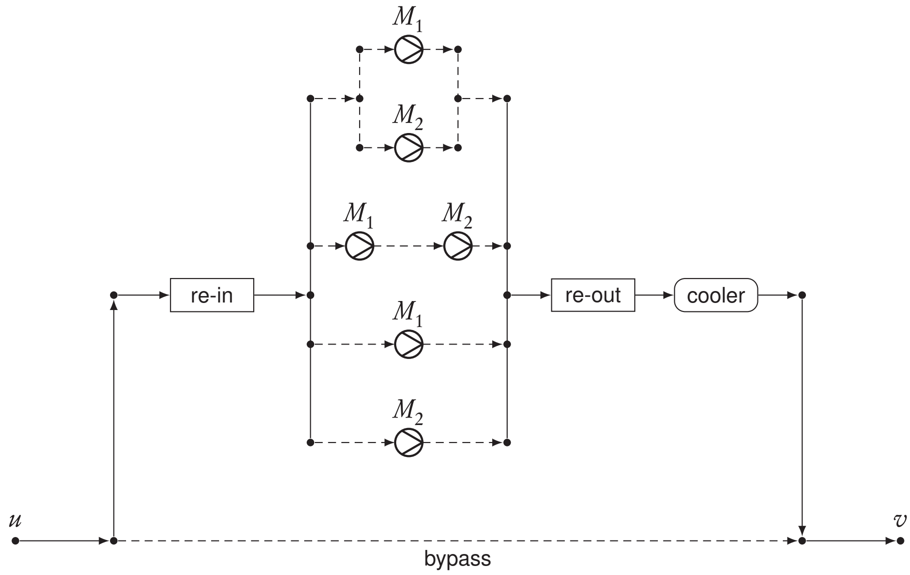


Figure 2.8. *A schematic plot of a compressor group with two machines, all possible configurations, and the corresponding bypass mode. The gas flow follows at most one of the dashed routes, i.e., either the bypass without any pressure loss, or one of the compressor configurations, in which case there is some pressure loss due to piping which is modeled by additional resistors.*

abstract modeling component that contains at least one control valve and additional network elements such as valves, preheaters, and resistors.

2.4.2 ▪ Compressor groups and configurations

In the following two sections, we describe subnetworks that contain compressors. The literature often uses the term compressors or compressor stations. These terms are sometimes used synonymously and sometimes not. In this section, we describe what we mean by a compressor group and later describe how we build a second layer of aggregation—what we then call a compressor station.

Small groups of compressor machines and drives are often used together in such a way that the gas enters through a single pipe, is routed through some of the compressor machines, and leaves via a single pipe. To model this, we introduce so-called compressor groups; see Figure 2.8. These entities encapsulate a set of compressor machines (together with the corresponding drives) that can be operated in different predefined ways. In practice it is often the case that one machine per group is specified as a so-called *backup compressor*. This compressor is not used in planning scenarios and is only specified for safeguarding against failures. In our categorization, compressor groups are active elements,

Figure 2.9. *The compressor station in Waidhaus, Germany. (Source: OGE.)*

i.e., network operators can control the *operation mode* of the groups. Technically possible modes are *active*, *closed*, and *bypass*, similar to control valves. We first describe the closed and bypass modes and then concentrate on the more complicated active mode.

If a compressor group is closed, it behaves like a closed valve. Thus, the gas flow is zero, and inflow and outflow gas states (i.e., gas pressure, temperature, and density) are decoupled. In bypass mode, the gas flows *around* the group and is therefore not affected by any part of the group. Since the group piping that bypasses the machines is very short, no significant friction effects occur, similar to (control) valves. In what follows, we neglect these insignificant friction effects completely and assume that inflow and outflow gas states are equal (see also Section 2.3.3).

If the compressor group is active, internal parallel or serial combinations of active compressors can be chosen. As a rule of thumb, parallel arrangements are capable of compressing a larger amount of gas, whereas serial combinations yield higher compression ratios. Due to technical limitations, not every arrangement of this type is possible, but the network operators can choose a finite set of arrangements, called *configurations*. In contrast to the bypass mode, the inner group piping may lead to a significant pressure drop, which we model by additional up- and downstream resistors as for control valves (see Section 2.3.4).

2.4.3 ▪ Compressor stations and subnetwork operation modes

Frequently, collections of compressor machines and valves are connected to more than two pipelines and may be used in various ways to route gas from some of those pipelines to other ones, increasing or even decreasing the pressure as necessary. Compressor groups are not sufficient to model such complex structures. Instead, for each such *compressor station*, an explicit subnetwork that reflects all possible routes of gas through the compressor station is required. Compressor stations are often located at intersections of several pipelines, and they are also used to route the gas between the connected pipeline systems; see Figure 2.9 for an example.

Similar to compressor groups, compressor stations internally allow for multiple paths that the flow of gas can actually take. The desired path is again selected by switching a cascade of individual valves, compressor groups, and (sometimes) control valves in the right way. This is carried out by human dispatchers, who are in charge of controlling the network's active elements. Their objective is to maintain a feasible flow of gas that ideally requires a minimum amount of energy, mainly consumed by active compressors. For this, valves can be opened or closed. Compressor groups and control valves can be activated, bypassed, or closed. If they are active, the level of operation and the configuration has to be chosen.

As we have already discussed for compressor groups in Section 2.4.2, some potentially possible switching combinations are not technically possible or not practically meaningful. In fact, only a very small subset of states might be relevant for operational purposes. The relevant states of the subnetwork can be represented by either an *inner* or an *outer* description. An inner description is a finite list of all allowed states for the controllable network elements of the subnetwork, where for each state of the subnetwork and each controllable element it is specified whether this element is either open (or active) or closed (or inactive) in this state. If the element is open, then additional bounds on the flow can optionally be specified. This option is typically used to specify the flow direction. An outer description is a list of linear constraints that describe the interdependency of active elements, for example, "if element A is open, then B is open, and C is closed," or "one of the elements A, B, C must be open, but not all three together."

Both descriptions are used in practice. From a polyhedral (although not from an algorithmical) point of view they are in fact equivalent: It is always possible to determine the outer description, if the inner is given, and vice versa. In fact, the inner description specifies a finite set of points (control decisions) $s^1, \ldots, s^k \in \{0,1\}^n$, where $s^i_j = 0$ represents the fact that the controllable network element j is closed in state i, and $s^i_j = 1$ represents that it is open (or active). The convex hull of these points, $Q = \operatorname{conv}\{s^1, \ldots, s^k\}$, is a bounded polyhedron, i.e., a polytope. The theorem of Minkowski and Weyl states that Q can alternatively be described as $\{x \in \mathbb{R}^n \mid Ax \leq b\}$ for some $A \in \mathbb{R}^{m \times n}$ and $b \in \mathbb{R}^m$, i.e., by finitely many linear inequalities given by the rows of this system; see, e.g., Nemhauser and Wolsey (1988); Ziegler (1994). This second form gives an outer description by linear constraints.

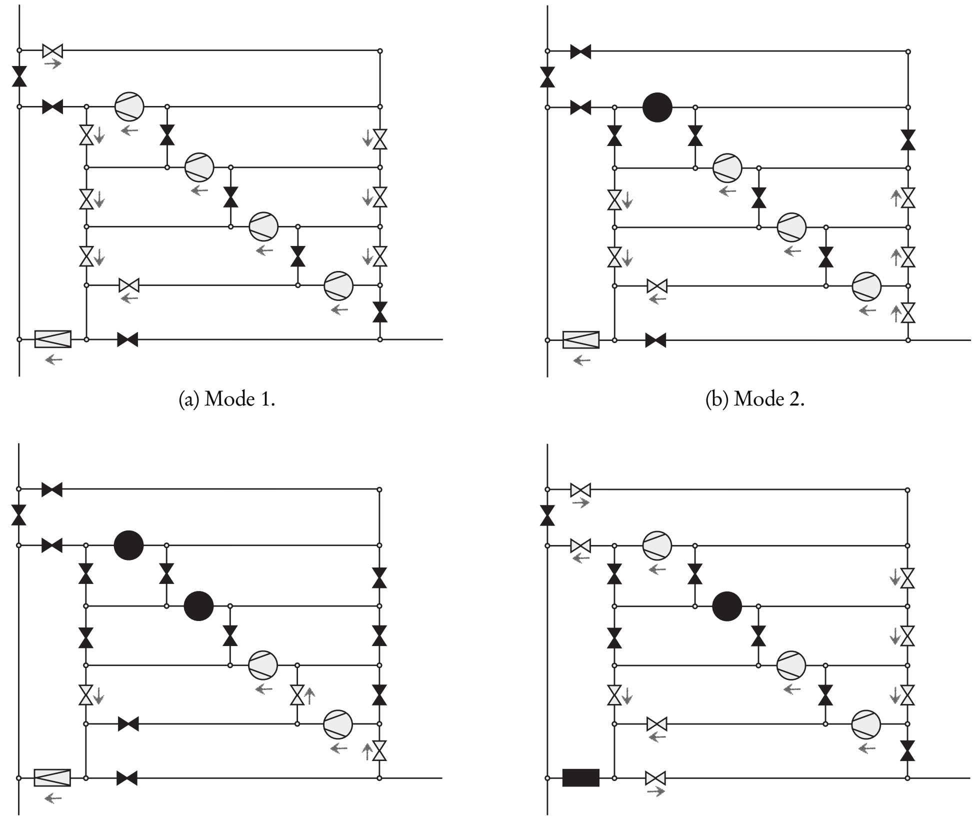
Note that the two descriptions can drastically differ in size. For example, the standard cube $[0,1]^n$ is the convex hull of 2^n points (and not fewer), but can be described by $2n$ linear inequalities. A notoriously open question is whether there exists an algorithm that converts between the two descriptions in time that is polynomial in the combined size of the input and output data; see, e.g., Avis, Bremner, and Seidel (1997).

Note that an outer description $Ax \leq b$, specified by rules on active elements as mentioned above, is usually *not* tight in the sense that in general

$$\operatorname{conv}\{x \in \{0,1\}^n \mid Ax \leq b\} \subsetneq \{x \in \mathbb{R}^n \mid Ax \leq b\}.$$

However, since the set on the left-hand side is obviously a polytope, there exists a (tight) linear description of it. This tighter formulation often has computational advantages, when available.

The conclusion is that active elements no longer appear individually, but are combined in blocks, and the decision on one element of such a block affects several other elements belonging to the same block. In this sense, the operational decisions (modes) on the state of the elements of the subnetwork are combined, and we thus speak of *subnetwork operation modes*. Figure 2.10 shows four examples of different subnetwork operation modes;



(a) Mode 1. (b) Mode 2.

(c) Mode 3. (d) Mode 4.

Figure 2.10. *Four different subnetwork operation modes for a large compressor station. Each of them leads to a different flow of the gas through the station—elements colored dark are "closed." Mode 1: Gas flow from north to south, quad parallel compression. Mode 2: Flow from east to south, triple parallel compression. Mode 3: Flow from east to south, double serial compression. Mode 4: Flow from north to east with double parallel and to south with single compression.*

see Table 2.1 for the symbols representing the different components. Each of the four modes leads to a different flow of the gas through the station.

We remark that subnetwork operation modes are not only used to model feasible states of compressor stations, but also other subnetworks that occur outside of physical stations. For example, groups of valves that are scattered over a large distance along parallel pipelines with joints can act as a subnetwork. This allows the operator to use these parallel pipelines as loops over a certain distance.

The operational knowledge in addition to the network's physical structure needs to be encoded in subnetwork operation modes, so that it becomes a useful representation of reality and it can be used in the models of the subsequent chapters.

2.5 ▪ Gas network representation

The different elements, groups, and stations as discussed in the previous sections appear as elements in gas networks. In this section, we introduce the representation of a gas network in terms of a directed finite graph $G = (V,A)$ with nodes V and arcs A. This graph representation will be used throughout the rest of this book and provides a basis for the derivation of the mathematical models for the problem of the validation of nominations in Chapters 6–10. Note that G will not contain self-loops, but parallel arcs might occur.

As a graph general notation, we use $\delta^-(u) := \{a = (v,u) \in A\}$ and $\delta^+(u) := \{a = (u,v) \in A\}$ for the set of incoming and outgoing arcs of $u \in V$, respectively. The set of all incident arcs is $\delta(u) := \delta^-(u) \cup \delta^+(u)$.

The set of entries is denoted by V_+ and the set of exits is denoted by V_-. All other nodes, i.e., junctions of network elements that are neither exits nor entries, are collected in the set V_0. Thus, the node set of a directed graph of a gas network is the disjoint union of these three sets, i.e., $V = V_+ \dot{\cup} V_- \dot{\cup} V_0$; see also Section 2.1.

The set of arcs $A = A_{\text{passive}} \dot{\cup} A_{\text{active}}$ consists of the set A_{passive} of arcs representing passive network elements and the set A_{active} of arcs representing active network elements. Pipes (Section 2.3.1) and resistors (Section 2.3.2) are passive network elements. The set of pipes is denoted by A_{pi}, and A_{rs} denotes the set of resistors. The set of resistors is further subdivided into the set $A_{\text{nl-rs}}$ of resistors causing a nonlinear pressure drop and the set $A_{\text{lin-rs}}$ of resistors causing a fixed pressure reduction in flow direction, i.e., $A_{\text{rs}} = A_{\text{nl-rs}} \dot{\cup} A_{\text{lin-rs}}$. Another type of passive network element, that is, only used for modeling purposes and thus not mentioned so far, is a so-called *short cut*. Short cuts can be thought of as very short pipes not causing any pressure reduction. We denote the set of short cuts by A_{sc}. For further details on short cuts we refer the reader to Section 5.1.3. Altogether, we have $A_{\text{passive}} = A_{\text{pi}} \dot{\cup} A_{\text{rs}} \dot{\cup} A_{\text{sc}}$.

The set of active network elements is made up of valves (Section 2.3.3), control valves (Section 2.3.4), and compressors (Section 2.3.5). Arcs representing valves are elements of the set A_{va}. The set of control valves is given by A_{cv}, and by A_{cm} we denote the set of compressor machines. Therefore, we have $A_{\text{active}} = A_{\text{va}} \dot{\cup} A_{\text{cv}} \dot{\cup} A_{\text{cm}}$. Finally, the set of control valves is further subdivided into the set $A_{\text{cv}}^{\text{aut}}$ of (automated) control valves equipped with a remote access device and the set $A_{\text{cv}}^{\text{man}}$ of manually operated (nonautomated) control valves. Thus, $A_{\text{cv}} = A_{\text{cv}}^{\text{aut}} \dot{\cup} A_{\text{cv}}^{\text{man}}$.

The mass flow q on an arc $a \in A$ is denoted by q_a. For the pressure p at a node $u \in V$, we write p_u. Other quantities associated to a certain arc or node of the networks are indexed by respective subscripts in the same way. The sign of q_a depends on the direction of the arc; i.e., if gas flows from node u to node v on an arc $a = (u,v)$, we have $q_a > 0$, and if gas flows from node v to node u, we have $q_a < 0$.