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Efficient Stochastic Programming Techniques for Electricity Swing Options

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EFFICIENT STOCHASTIC PROGRAMMING TECHNIQUES FOR ELECTRICITY SWING OPTIONS

MARC C. STEINBACH AND HANS-JOACHIM VOLLBRECHT

ABSTRACT. We consider the valuation of contracts of electrical energy supply with optionalities. After discussing appropriate stochastic programming models and presenting especially suited solution algorithms, a set of price scenarios is simulated based on a probabilistic model of the electricity spot market price at the EEX. We determine empirically upper and lower bounds for the stochastic optimization over any scenario tree obtained by reduction techniques. Furthermore, we introduce constraints restricting all scenarios to have identical contract exercise amounts cumulated over various fixed subperiods. Calculation of the losses of the optimal value of the objective function caused by these constraints shows that, for subperiods of one month, no substantial loss is encountered. This suggests a temporal decoupling heuristic where the depth of scenario trees is reduced to a suitable subperiod, yielding a good approximation to the valuation problem with substantially reduced complexity.

1. INTRODUCTION

Due to the liberalization of energy markets, the trading of energy plays an increasingly important role in planning tasks of energy producers and distributors. In the case of electric energy, liquid spot markets have come into existence, so that the economic value of a decision can be reduced to the stock exchange value of a corresponding product. Decision makers are thus faced with the necessity of developing and solving stochastic programming models, with the stochastic process of electricity prices as a key component. Being discrete in time and value, this process can be modeled by means of scenario trees—however, trees of astronomical size. This raises the important and difficult question how to approximate the process by scenario trees of tractable size. In one type of approach [13], the tree is constructed from analytical price models, such as an adapted Pilipovic model [24, 25], possibly combined with bounding techniques to give upper and lower estimates of the optimal value. Another type of approach constructs trees from simulated scenarios by a recursive reduction technique that controls the approximation quality using suitable metrics of distributions and filtrations [16]. In the study presented here, an initial scenario fan has been generated by simulation from a price model based on [26], which includes deterministic influences (trend and seasonal periodicities) and stochastic influences (outliers and residuals, using an ARMA model), combined by multiplication.

The particular optimization problem addressed in this study is the valuation of certain swing options [4] in which the option holder may exercise, within given limits, variable energy amounts from a fixed total contract volume during a given period. Flexible contracts of this kind originate in the natural gas industry [1]; nowadays they are commonly used as hedging instruments in power and energy markets and are even being considered for pricing IT resources [7].

Valuation schemes for swing options have been developed systematically during the last decade. Most of them are based on or closely related to stochastic dynamic programming (SDP), often combined with special techniques that exploit simplifications resulting from the structural properties of specific price models or specific classes of swing options. A widely known method is the Least Squares Monte Carlo algorithm [22], which approximates conditional expectations within the SDP scheme by least squares estimates. Related Monte Carlo simulation approaches are proposed in [18] and [23]. A quantization approach (optimal discretization) is proposed in [2] and combined with a decomposition of the payoff function of certain swing options for highly accurate valuation under multi-factor Gaussian price processes. In [9], a specific Poisson price model is developed deriving the probability for spikes from the electricity supply/demand ratio, and analytical price formulas are given for very simple weekly swings. In [20], electricity swing

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options are modeled as portfolios of forwards and call options, and a lower bound on their value is derived by linear optimization under the assumption of a Markov consumption process. In [6, 5], the valuation problem is formulated and analyzed as an optimal multiple stopping problem for price processes in the form of geometric Brownian motion and general linear regular diffusion, respectively. A related formulation as stochastic impulse control problem is analyzed in [8], together with a numerical algorithm for the corresponding Hamilton–Jacobi–Bellman quasi-variational inequality. Other valuation approaches employ direct discretizations of the price process in space and time within an SDP scheme: grid discretizations are used in [32] and [15], whereas forests are used in [21] and [19]. Finally, a scenario tree model is proposed in [14], and special aggregation and reparameterization techniques are developed to obtain a multistage stochastic linear program tractable by state-of-the art optimization software like CPLEX. The multistage stochastic programming approach is the most flexible and general one as it does not impose special structural requirements on price processes or constraints of the swing option.

Many of the above references address the peculiarities of price processes in energy (specifically electricity) markets, which are characterized by mean-reversion, jumps, and spikes. For highly realistic forward price models involving these phenomena, regime switching, and stochastic volatilities, see also [10, 11].

We consider a stochastic programming model for valuating general swing options, and demonstrate that our existing optimization algorithms perform very well on that problem class. This holds regardless of a specific price model or scenario tree. Next we aim at constructing scenario trees by reduction in a way that is especially suited for the valuation problem, starting from a fan of price scenarios. We wish to develop techniques that help deciding on when to branch and how many branches to use. The approach taken here is based on the idea of using the *value of the stochastic solution* (VSS). The VSS provides information on any scenario tree obtainable from the given fan, since the fan (where branching occurs only at $t = 0$) represents near-perfect information: the complete process of future prices is already determined at $t = 1$. In the valuation problems to be considered, the VSS depends mainly on

(1) the total option volume;
(2) the contract period (planning horizon).

The first dependence is non-monotonic since the VSS measures the value of additional flexibility of a stochastic exercise strategy over a deterministic one. This flexibility decreases when the total option volume approaches the maximal amount within exercise limits. We evaluate the VSS numerically over the range of possible option volumes.

The second dependence is clearly monotonic: the VSS will increase with the planning horizon. As swing options usually have long contract periods, we investigate the effects of a temporal decomposition into subperiods where the cumulated energy amounts are required to be identical for all scenarios. The extra constraints will reduce the flexibility (hence the VSS), but for the price model considered we expect a moderate loss only. This is confirmed by our study, even if we distribute the total energy a priori over suitably chosen subperiods.

The paper is organized as follows. We formulate the general valuation problem in Sect. 2, discuss some basic properties and present our solution technology. The concrete problem and price model are given in Sect. 3. Three computational experiments investigating the VSS are described in Sect. 4, with results reported in Sect. 5. In Sect. 6 finally, we discuss the results and outline an approximation heuristic to determine the timescale and structure of tree branchings under suitable assumptions. This heuristic reduces the complexity of the valuation problem without significant loss of the approximation quality.

2. General valuation problem

In setting up the mathematical formulation of the valuation problem, we follow [14] where more details can be found. We consider a planning horizon $[0, T]$ divided into periods $t = 1, \dots, T$ of unit length, where $[0, T]$ represents the (remaining) contract period and each subinterval represents an exercise period. During period t, the option holder may exercise any power p_t within an agreed range $\mathcal{P}_t = [p_t^-, p_t^+]$. Often there are additional ramp constraints (or *ratchets*), limiting the power difference $r_t = p_t - p_{t-1}$ to some range $\mathcal{R}_t = [r_t^-, r_t^+]$. Finally, the cumulated energy e_t up to time t may vary within $\mathcal{E}_t = [e_t^-, e_t^+]$. In practice, the bounds $p_t^\pm$ will typically be constant or change just a few times over $[0, T]$, ratchets will usually have the form $r_t^\pm = \pm\rho$ with ρ fixed, and energy limits $e_t^\pm$ will only be specified at T and possibly a few more time instances.

Further, let K denote the strike price of the option, s_t the spot price during period t, $k_t := K - s_t$, and e_0, p_0 the initial energy and power values at $t = 0$. Here p_0 will only be relevant in the presence of ratchet constraints, and we have $e_0 = 0$ if the planning period coincides with the entire contract period.

2.1. **Stochastic programming model.** Our probabilistic model is based on a finite number of scenarios with associated probabilities, organized as a scenario tree. Let V denote the set of nodes (vertices) of the tree, τ_j the probability of node j, $L_t \subseteq V$ the level set of nodes at depth t, and L the set of leaves; further $1 \in L_1$ the root, $j \in L_t$ the "current" node, $S(j)$ its set of successors, $i \equiv \pi(j)$ its unique predecessor (if $t > 1$), and $\Pi(j) = \{1, \dots, i, j\}$ the unique path from the root to j. Finally define $V^* := V \setminus \{1\}$. The subtree rooted in j has respective vertex set, level sets and leaves $V(j)$, $L_t(j)$, and $L(j)$. Below, the vertex set is often taken to be $V = \{1, \dots, N\}$ where nodes are numbered in any ascending order. (In the deterministic case we have $V = \{1, \dots, T\}$.)

More concretely, the stochastic price process (s_t) has discrete realizations s_j defining the scenario tree and inducing realizations e_j, p_j, r_j, k_j of e_t, p_t, r_t, k_t. We define $\pi(1) = 0$ with e_0, p_0 as above, and variable vectors $r, p, e \in \mathbf{R}^{|V|}$ with associated feasible sets $\mathcal{R}_V, \mathcal{P}_V, \mathcal{E}_V$, where $r = (r_j)_{j \in V}$ and $\mathcal{R}_V = \prod_{j \in V} \mathcal{R}_j$, etc.

The general valuation problem then becomes a stochastic LP,

$$\begin{aligned} \underset{r,p,e}{\text{Minimize}} \quad & \sum_{j \in V} \tau_j k_j p_j && (1) \\ \text{subject to} \quad & p_j = p_{\pi(j)} + r_j, \qquad j \in V, && (2) \\ & e_j = e_{\pi(j)} + p_j, \qquad j \in V, && (3) \\ & (r, p, e) \in \mathcal{R}_V \times \mathcal{P}_V \times \mathcal{E}_V. && (4) \end{aligned}$$

This is a multistage model in control form, with "incoming control" in the terminology of [31]. This means that the state variables p_j, e_j depend on the control variable r_j of the *same* node, reflecting the assumption that the decision for period t is made *after* observing the actual price s_j for period t, at time $t - 1$. Concretely, we control the exercise process via the power differences r_t; the exercised power p_t is the sum of these differences (first integral), and the cumulated energy e_t is the sum of the powers (second integral).

In the absence of ratchets, one obtains a simplified stochastic LP model,

$$\begin{aligned} \underset{p,e}{\text{Minimize}} \quad & \sum_{j \in V} \tau_j k_j p_j && (5) \\ \text{subject to} \quad & e_j = e_{\pi(j)} + p_j, \qquad j \in V, && (6) \\ & (p, e) \in \mathcal{P}_V \times \mathcal{E}_V. && (7) \end{aligned}$$

Again we have incoming control form, now with states e_j and controls p_j. Thus we control the exercise process with the power p_t, and obtain the cumulated energy e_t as first integral.

Throughout the paper we let $K := 0$; hence minimizing $\sum \tau_j k_j p_j$ is the same as maximizing $\sum \tau_j s_j p_j$, the expected spot market value of the contract.

2.2. **Aggregated formulation and critical prices.** The two models above represent the most straightforward and most flexible formulations, in that branchings of the scenario tree and bounds on all variables (including cumulated energy) are allowed at each node. In practice, energy bounds will only be present at a few points. Moreover, due to the large number of decision periods, branchings of the scenario trees will occur at relatively few time instances only. In this case every tree node j may represent several periods, $t_{\pi(j)} + 1, \dots, t_j$, where $t_{\pi(1)} = 0$ and $t_j = T$ for $j \in L$. Each vector of node variables holds only a single energy value (associated with t_j): $(r_j, p_j, e_j) \in \mathbf{R}^{2d_j+1}$ or $(p_j, e_j) \in \mathbf{R}^{d_j+1}$, where $d_j := t_j - t_{\pi(j)} - 1$.

Letting $\mathbf{1}_j := (1, \dots, 1) \in \mathbf{R}^{d_j}$, the stochastic LP models can now be reformulated with aggregated periods:

$$
\begin{aligned}
\underset{r,p,e}{\text{Minimize}} \quad & \sum_{j \in V} \tau_j k_j^T p_j && && (8)\\
\text{subject to} \quad & (I - N_j) p_j = M_j p_{\pi(j)} + r_j, && j \in V, && (9)\\
& e_j = e_{\pi(j)} + \mathbf{1}_j^T p_j, && j \in V, && (10)\\
& (r, p, e) \in \mathcal{R}_V \times \mathcal{P}_V \times \mathcal{E}_V && && (11)
\end{aligned}
$$

and

$$
\begin{aligned}
\underset{p,e}{\text{Minimize}} \quad & \sum_{j \in V} \tau_j k_j^T p_j \\
\text{subject to} \quad & e_j = e_{\pi(j)} + \mathbf{1}_j^T p_j, \qquad j \in V,\\
& (p, e) \in \mathcal{P}_V \times \mathcal{E}_V.
\end{aligned}
$$

Here $N_j \in \mathbf{R}^{d_j \times d_j}$ contains unit entries on the lower secondary diagonal and zeros elsewhere, while $M_j = \mathbf{1}_j \mathbf{1}_{\pi(j)}^T \in \mathbf{R}^{d_j \times d_{\pi(j)}}$, containing a single unit entry in the upper right corner. These formulations reduce the total numbers of variables to roughly two thirds and one half, respectively.

It is well-known that the LP without ratchets has a simple solution in the deterministic case: there exists a critical price s^* such that $p_t = p_t^-$ is optimal whenever $s_t < s^*$, and $p_t = p_t^+$ is optimal whenever $s_t > s^*$. (The optimal power for $s_t = s^*$ depends on the actual energy bounds.) In fact, the critical price is given by the optimal dual variables, $s^* = K - y = K - w^- - w^+$. The aggregated formulation just introduced yields a direct generalization of this result to the stochastic case. From the Lagrangian

$$
\begin{aligned}
L(p, e, y, v^\pm, w^\pm) \quad = \quad \sum_{j \in V} \tau_j \Big[& k_j^T p_j - y_j (e_{\pi(j)} + \mathbf{1}_j^T p_j - e_j) \\
& - (v_j^-)^T (p_j - p_j^-) - w_j^- (e_j - e_j^-) \\
& - (v_j^+)^T (p_j^+ - p_j) - w_j^+ (e_j^+ - e_j) \Big]
\end{aligned}
$$

we obtain the dual LP

$$
\begin{aligned}
\underset{y, v^\pm, w^\pm}{\text{Maximize}} \quad & -e_0 y_1 + \sum_{j \in V} \tau_j \left[(p_j^-)^T v_j^- + e_j^- w_j^- - (p_j^+)^T v_j^+ - e_j^+ w_j^+ \right] \\
\text{subject to} \quad & \mathbf{1}_j^T y_j = k_j + v_j^+ - v_j^-, \qquad j \in V,\\
& y_j = w_j^- - w_j^+, \qquad j \in V,\\
& \tau_j y_j = \sum_{k \in S(j)} \tau_k y_k, \qquad j \in V \setminus L,\\
& v^\pm, w^\pm \geq 0.
\end{aligned}
$$

Starting with $y_j = w_j^- - w_j^+$ for $j \in L$, the remaining multipliers y_j are thus determined recursively as expectations of their successor variables,

$$
y_j = \sum_{k \in S(j)} \frac{\tau_k}{\tau_j} y_k = \sum_{k \in L(j)} \frac{\tau_k}{\tau_j} (w_k^- - w_k^+).
$$

This defines a martingale process on the scenario tree, which induces a *martingale process of critical prices* $s_j^* = K - y_j$. In other words: along every branch of the scenario tree, the optimal exercise strategy is as in the deterministic case, just with different critical prices. Of course, the same holds in the non-aggregated model for every node sequence without branching.

Note finally that one can rewrite the *implicit* transition equation (9) of the aggregated problem with ratchets as an *explicit* one, $p_j = G_j p_{\pi(j)} + E_j r_j$, with $E_j := (I - N_j)^{-1}$ containing unit entries in the lower triangle and zeros else, and $G_j := (I - N_j)^{-1} M_j = E_j M_j$ containing unit entries in the last column and zeros else. All the LP variants above can thus be written in the following incoming control form, which is

TABLE 1. Computational statistics for concrete valuation problem on one processor of a 2.67 GHz Intel Core 2 E6700; relative accuracy 10^{-10} (duality gap + residuals)

Stochastic LP	without ratchets	with ratchets
Number of rows	4’415’001	8’830’002
Number of columns	8’830’002	13’245’003
Number of nonzeros	13’245’002	30’905’004
Percentage of nonzeros	3.4×10^{-5}	2.6×10^{-5}
Order of KKT system	13’245’003	22’075’005
KKT vector memory	101.1 MB	168.4 MB
KKT factor memory	101.1 MB	202.1 MB
Total process memory	1181 MB	1859 MB
Number of iterations	89	77
Total solution time	219 s	288 s
Time per iteration	2.46 s	3.74 s

a special case of the tree-sparse problems considered in [31]:

$$\begin{aligned} \underset{u,x}{\text{Minimize}} \quad & \sum_{j\in V} \tau_j(d_j^T u_j + f_j^T x_j) \\ \text{subject to} \quad & x_j = G_j x_{\pi(j)} + E_j u_j + h_j, \qquad j \in V, \\ & (u,x) \in [u^-,u^+] \times [x^-,x^+]. \end{aligned}$$

2.3. **Stochastic programming algorithms.** In solving the valuation problems above, the main difficulty is the potentially excessive size of scenario trees. Our tree-sparse solution approach is based on the general idea of tackling the problem by iterative optimization methods that preserve the overall sparse structure in the linear subproblems, so that the latter are solvable by highly efficient linear algebra. The idea goes back to trajectory optimization problems [27, 28] and has been extended to stochastic optimization in [29, 30, 31]. Specifically, we use a primal-dual interior point method in connection with a factorization of the large tree-sparse KKT system defining the Newton step in each iteration. The KKT system can be interpreted as a linear-quadratic regulator problem defined over the scenario tree, with additional local and global constraints arranged into a generic hierarchical scheme. The associated factorization can be interpreted as a hierarchical sequence of projections in each node, combined with a dynamic programming recursion over the tree. Assuming that the number of variables in all nodes is comparable, this algorithm achieves optimal complexity $O(|V|)$ with respect to both memory and runtime.

To exploit application-specific structural properties, a software tool has been developed [17] that analyzes the specific KKT structure and generates a custom factorization as source code in C++. Data structures and operations of the custom implementation are designed with the aim of minimal storage requirements and moderate operation counts. This approach is particularly efficient on regularly structured models like the one under consideration. For instance, the constraints matrix contains only unit entries (±1) in fixed positions. Consequently, the custom implementation can represent the matrix entirely in code. It needs no memory to store entries or even entry positions. Similarly, the KKT matrix factorization has many such entries which are detected and represented in code; storage will only be allocated for the remaining entries, along with code that computes their values. Table 1 provides computational statistics for the solution of the concrete valuation problem with and without ratchets, using a scenario fan with 1000 scenarios (see below). Observe that each problem is solved in less than 5 minutes on a 2 GB machine, with total memory (code and data) less than 12 times the size of one KKT vector. Note also that the per-iteration effort depends only on the size of the scenario tree but not on its topology.

Table 2 gives a comparison of runtime and storage requirements of our solver IPM/TreeKKT with ILOG CPLEX 10.1.1 (dual simplex and barrier) on the valuation problem with ratchets for various numbers of scenarios. Both solvers run on the same machine but under different operating systems: CPLEX under Windows XP Professional, and IPM/TreeKKT under GNU/Linux (openSuSE 10.3). CPLEX reads an LP file and then solves the problem after some preprocessing; IPM/TreeKKT only reads the problem dimensions and price scenarios and then solves the problem without any preprocessing. The additional storage and memory

TABLE 2. Computational comparison for concrete valuation problem on one CPU of a 2 GHz Intel Xeon 5130 server with 12 GB RAM: IPM/TreeKKT vs. CPLEX 10.1.1

Solution time in seconds: (read data) + presolve + solve

Scenarios	IPM/TreeKKT	CPLEX dual simplex	CPLEX barrier
100	$(0.3) + 0 + 30.2$	$(8.3) + 2.1 + 13.3$	$(8.3) + 2.3 + 39.3$
200	$(0.6) + 0 + 62.1$	$(51.6) + 4.2 + 34.6$	$(70.6) + 4.6 + 73.6$
300	$(0.8) + 0 + 97.0$	$(92.5) + 6.3 + 63.9$	$(109.8) + 7.0 + 121.1$
500	$(1.4) + 0 + 171.7$	$(66.0) + 10.9 + 127.4$	$(66.8) + 11.6 + 218.7$
1000	$(2.7) + 0 + 401.1$	$(243.4) + 21.4 + 453.1$	$(237.9) + 23.5 + 529.9$

Total memory in MB: (read data) + solve

Scenarios	IPM/TreeKKT	CPLEX dual simplex	CPLEX barrier
100	$(0) + 213$	$(170) + 581$	$(170) + 705$
200	$(0) + 397$	$(319) + 1192$	$(319) + 1410$
300	$(0) + 580$	$(481) + 1791$	$(481) + 2130$
500	$(0) + 946$	$(700) + 3240$	$(700) + 3810$
1000	$(0) + 1862$	$(1940) + 6007$	$(1940) + 7180$

requirements for reading the LP files into CPLEX (in parentheses) are mainly shown for completeness: they can probably be reduced to the corresponding values of IPM/TreeKKT by using the CPLEX callable library interface, and will be disregarded in the following discussion.

The runtime comparison shows the typical behavior of interior point methods and (dual) simplex for an LP. The simplex algorithm clearly outperforms our interior point method on smaller problem instances. For 100 scenarios it is nearly twice as fast, with 15.4 s versus 30.2 s. Conversely, our interior point method performs better on large instances, with 401.1 s versus 474.5 s on the 1000 scenario problem. The CPLEX interior point method (barrier) behaves similar in principle but is between 26% and 38% slower than our algorithm.

The main advantage of IPM/TreeKKT becomes obvious in the memory comparison. A linear interpolation for $n = 500$ and $n = 1000$ scenarios yields as memory requirements $30 + 1.832n$ for IPM/TreeKKT, $473 + 5.534n$ for CPLEX dual simplex, and $440 + 6.74n$ for CPLEX barrier. This is exact for IPM/TreeKKT (up to rounding errors) while CPLEX actually needs less memory on the smaller instances. In any case, on the large instances CPLEX dual simplex and CPLEX barrier require 3.0 times and 3.7 times more memory per scenario than IPM/TreeKKT, respectively. On the 12 GB server, the largest solvable instances thus have roughly 1650 and 2000 scenarios, versus 6300 with IPM/TreeKKT.

3. CONCRETE VALUATION PROBLEM

3.1. **Evaluated contract.** A bilateral contract with optionalities for obtaining electrical energy from an energy supplier has to be valuated. This valuation is based on its optimal exercise at the European Energy Exchange (EEX) spot market: the value of an exercise of the contract at a particular hour is interpreted as the corresponding value at the EEX spot market. The contract defines exercise rights for obtaining variable amounts of energy:

- contract period in h: $T = 4416$ (2 quarters Q1, Q2, 1.7.2006-31.12.2006)
- contract volume in GWh: $e^{\pm}_{2208} = 50$ (Q1), $e^{\pm}_{4416} = 240$ (Q2)
- power limits in MW: $\mathcal{P}_t = [0, 90]$ (Q1), $\mathcal{P}_t = [25, 145]$ (Q2)
- ratchets in MW: $r^{\pm}_t = \pm 60$

This type of contract can be viewed as a swing option [4]. With a swing option, the owner has the possibility to hedge his risks at the spot market with flexible exercise profiles of the option, which other hedging instruments such as futures are not capable of because of their rigid exercise structure allowing just to hedge with a fixed exercise that may differ only from peak to off-peak period. General swing options may require also temporal constraints such as a minimum period between two swings (changes in exercise). In our example, this is not required. For an example, Figure 1 shows part of an optimal exercise strategy for a simulated spot price scenario.

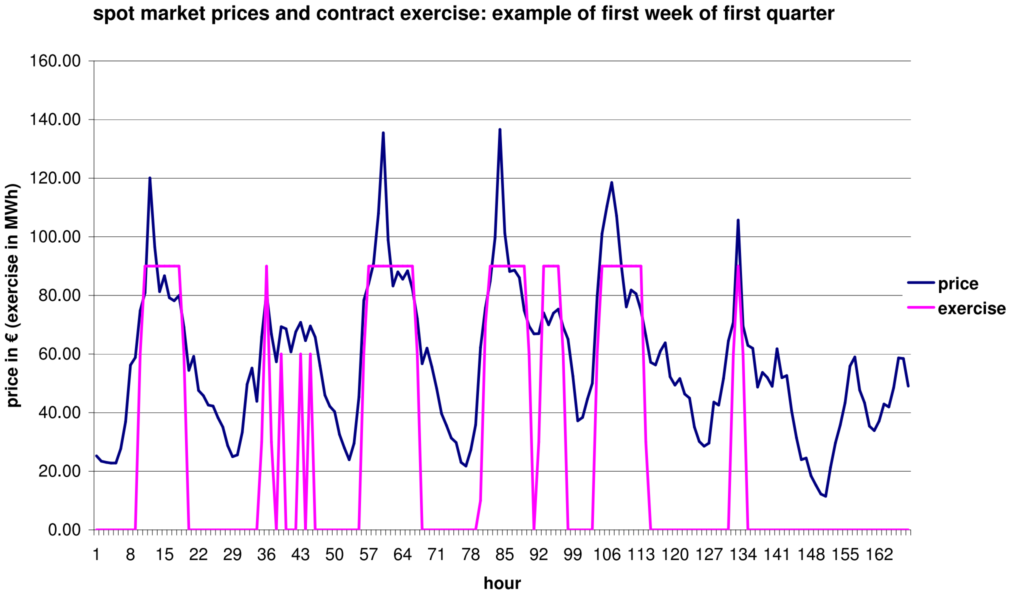

FIGURE 1. Example of an optimal exercise for a single scenario (partial view)

3.2. **Price model.** From the numerous electricity price models published so far, we have chosen the one described in [26]. Details and their justification can be found there. In this section we just sketch this model and outline where our model differs from it.

The spot price s_t is decomposed into four components: two deterministic ones, the trend s_t^{tr} and the seasonal part s_t^{seas}, and two stochastic ones, the outlier part s_t^{out} and the residual part s_t^{res}. The price s_t without outliers is modeled as

$$s_t = s_t^{tr} s_t^{seas} s_t^{res}. \tag{12}$$

The trend follows an exponential model; the seasonal part models daily profiles and the yearly seasons. The latter are modeled by a trigonometric polynomial with a basic oscillation of one year plus the first harmonic of half a year. For the daily profiles, we define five categories of days: Monday or day after or between holidays, Tuesday to Thursday, Friday or day before holiday, Saturday, Sunday or holiday. For each of these categories and for every hour (1–24), we estimate an independent model based on the trigonometric polynomial. The residuals are assumed to follow an ARMA process with time lags of $t-1$, $t-24$ and $t-25$. For estimating the parameters, we transform the historical residuals to obtain a normal distribution. Outliers are then added to the model by a random process that creates an outlier at t with probability p_t^{out}, depending on whether there has been an outlier at certain preceding points of time. The values of outliers are modeled by the gamma distribution. All probabilistic models are calibrated on historical data, specifically, EEX spot prices of the years 2003–2006.

3.2.1. *Scenario simulation.* We generate n price scenarios by Monte-Carlo simulation using the price model just presented. The scenarios are assumed to have identical probability $\tau_j = 1/n$. This yields a scenario fan as in Fig. 2, which represents the approximation of the price process that our experiments are based on (see Sect. 4). Note that the construction of a scenario tree according to [16] starts from such a scenario fan, to which certain reduction steps are applied. Note further that, with n in the range of tractability ($n = 1000$ in our experiments), the probability of obtaining a scenario fan rather than a more general tree is close to one since the probability of sampling two identical prices at $t = 1$ is almost zero. Figure 3 shows 30 simulated scenarios for a period of one week.

3.2.2. *Approximation with a tree or a fan?* This question motivates the experiments to be presented in the next section. In the following, we briefly discuss this item.

There are strong arguments for preferring general scenario trees (branching at any point of time) over a scenario fan (branching only at $t = 0$) [12, 4, 3]: in a stochastic process, chance unfolds all the time and not only at the very beginning. Optimization based on a fan works with an erroneous process model, since

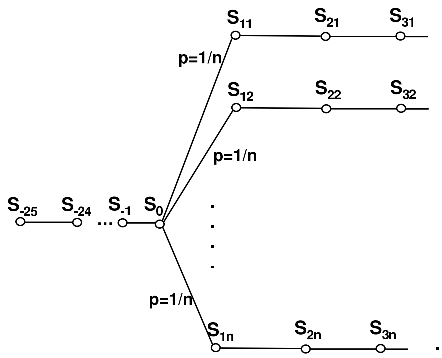

FIGURE 2. The simulated scenario fan with spot market prices

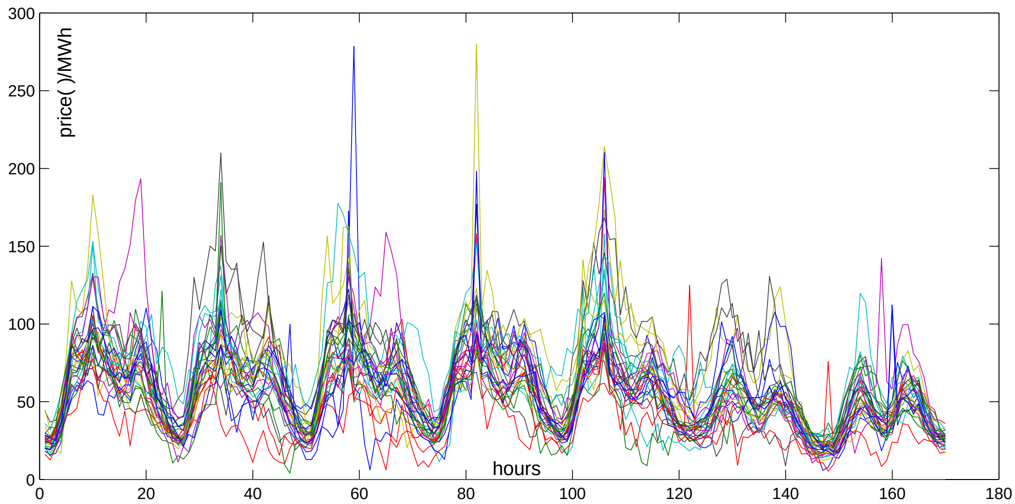

FIGURE 3. 30 simulated scenarios for a sample week

all decisions after the first step are deterministic (from $t = 1$ on, the future price process is completely known until the end of the contract period). This should lead to a result that is too optimistic since the algorithm has perfect information about the future.

On the other hand, the theory of scenario tree reduction [16] tells us that a tree constructed by reduction from a fan approximates the distribution and filtration given by that fan, where the reduction method guarantees stability in the sense that convergence of reduced scenario trees to the reference tree (fan) under a particular metric leads to convergence of the optimal values to the optimal value for the reference tree (fan).

It seems that the question whether a sampled fan (of moderate size) or a tree constructed by reduction from such a fan gives a better approximation of the optimal value cannot be answered in general, notwithstanding the strong structural argument given at the beginning of this paragraph in favor of a tree. For this reason, we perform three experiment series, to quantitatively evaluate the impact of the structural constraints that a tree introduces to the valuation problem.

4. COMPUTATIONAL EXPERIMENTS

4.1. **Experiment 1.** Using the 1000 scenarios just described, we solve the valuation problem (1–4) for the contract of Sect. 3.1 over the following range of contract volumes (in GWh):

$$e^{\pm}_{2208} = 50\alpha \in [0, 95], \quad e^{\pm}_{4416} = 55.2 + (240 - 55.2)\alpha. \tag{13}$$

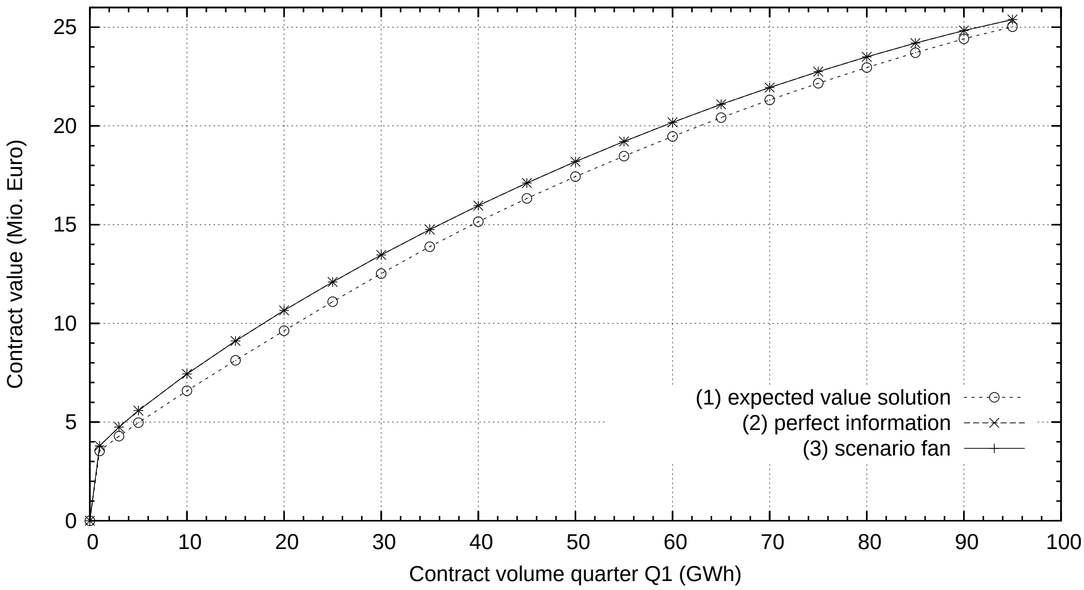

FIGURE 4. Optimal values for Experiment 1

Thus, α is a common scaling factor for the nominal exercise volumes of quarters Q1 and Q2 in excess of the base load $\sum_{\tau=1}^{t} p_{\tau}^{-}$. Three cases are distinguished: (1) *expected value:* deterministic optimization of the single scenario of expected prices, $\bar{s}_t = \sum_{j \in L_t} \tau_j s_j$; (2) *wait-and-see:* separate optimization of each scenario; (3) *here-and-now:* joint optimization over the scenario fan. The difference of the mean optimal value of (2) and the optimal value of (3) is called the *expected value of perfect information* (EVPI). For the given fan it turns out to be almost zero, indicating (as expected) that the nonanticipativity constraint at $t = 0$ (decisions being identical for all scenarios) has a negligible impact. The difference of the optimal values of (3) and (1) is called the *value of the stochastic solution* (VSS). This value is more interesting in our case. Results will be discussed in Sect. 5.1.

4.2. **Experiment 2.** Here we valuate the contract over individual scenarios with its nominal volume, $e_{2208}^{\pm} = 50$, $e_{4416}^{\pm} = 240$, but with additional inter-scenario volume constraints

$$e_j = e_k \quad \forall j, k \in L_t, \quad \forall t \in \mathcal{T}, \tag{14}$$

where $\mathcal{T} \in \{\mathcal{T}_d, \mathcal{T}_w, \mathcal{T}_m\}$ selects the set of final periods either of each day, week, or month within the contract period. Thus we require that the cumulated energy exercised during each of these subperiods agrees for all scenarios, while the respective amounts are subject to optimization.

Compared to experiment 1 we use a relatively small number of 30 scenarios. This is due to the fact that we do not have a suitable modeling environment for stochastic optimization with inter-scenario constraints, so we have to set up the deterministic equivalent manually. Consequently, the results of experiment 2 are not statistically significant. However, these results just serve for motivating experiment 3 which runs again on 1000 scenarios.

4.3. **Experiment 3.** Next we valuate the contract again over individual scenarios and with its nominal volume, but now with additional per-scenario volume constraints

$$e_j = \bar{e}_t \quad \forall j \in L_t, \quad \forall t \in \mathcal{T}, \qquad \mathcal{T} \in \{\mathcal{T}_d, \mathcal{T}_w, \mathcal{T}_m\}, \tag{15}$$

where $\bar{e}_t$ denotes the optimal cumulated volume from experiment 1, case 1. Thus we require again that the cumulated energy exercised during each subperiod agrees for all scenarios, but now with prescribed amounts.

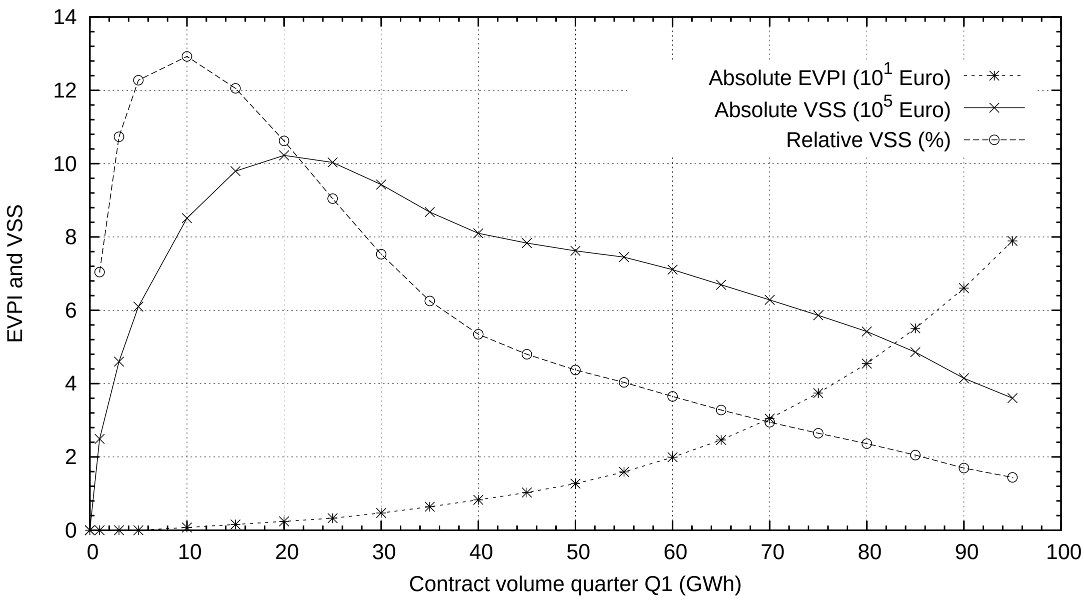

FIGURE 5. EVPI (absolute) and VSS (absolute and relative) for the scenario fan

5. COMPUTATIONAL RESULTS

5.1. **Experiment 1.** Figure 4 shows the optimal contract value for the three cases of experiment 1, in dependence of the contract volume of quarter Q1. It turns out that the factor α is bounded by the freely exercisable energy, $\sum_\tau (p_\tau^+ - p_\tau^-)$, during quarter Q2: the valuation problem becomes infeasible at

$$\hat{\alpha} = \frac{240 - 50 - 2208 \times 0.025}{2208 \times (0.145 - 0.025)} \gtrapprox 1.9656, \quad \text{or} \quad \hat{e}^\pm_{2208} \gtrapprox 98.2789. \tag{16}$$

We call $100 e^\pm_{2208} / \hat{e}^\pm_{2208}$ the *coverage*; it measures the percentage of hours of Q2 necessary to exhaust the contract volume when exercising p_t^+ during these hours, thus indicating a lack of flexibility. We will use $e^\pm_{2208}$ as a good approximation of the coverage.

Turning to Figs. 4–5, we confirm that the EVPI (difference of cases 2 and 3) is indeed very small (< 80 € throughout). Hence, we compare case 1 ("deterministic optimization") with cases 2, 3 ("stochastic optimization"). As can be seen from Fig. 5, the gain by stochastic optimization (VSS) increases rapidly for small coverage reaching an early maximum, and then decreases moderately toward large coverage (i. e., little flexibility in exercising the contract). The absolute VSS varies between 0.35 and 1 Million Euro, having its maximum at 20% coverage. The relative VSS reaches a maximum of 13% at 10% coverage, then decreases to 4% at about 50% coverage and to 2% at about 85% coverage.

Note that the optimal contract value for any scenario tree based on the same scenarios must lie between the values of cases 1 and 2; thus the VSS studied in Fig. 5 (plus the negligible EVPI) is an upper bound for the given set of scenarios.

5.2. **Experiment 2.** Next we consider the modified problem where inter-scenario constraints require the cumulated energy exercised during certain subperiods (day, week, or month) to agree for all scenarios. Subperiods of an hour would be equivalent to case 1 above (deterministic optimization), yielding a lower bound on the optimal value. The given subperiods must result in contract values between cases 1 and 2.

Table 3 gives the relative losses of the optimal contract value with respect to case 3 for the first quarter. The monthly constraint has very little influence on the value, and also the weekly constraint causes only a moderate loss. Therefore the variability that is exploited by perfect information hardly extends over more than a week. This qualitative interpretation is not surprising, however. It reflects the fact that, in our price model, the long-term behavior is modeled mainly by the *deterministic* components, trend and season. The stochastic component of outliers has little influence, and the residuals generate volatility only over a short

TABLE 3. Mean loss (in %, over first quarter) due to subperiod constraints

0.71	identical monthly amounts
1.77	identical weekly amounts
4.59	identical daily amounts
7.70	identical hourly amounts (deterministic optimization: case 1)

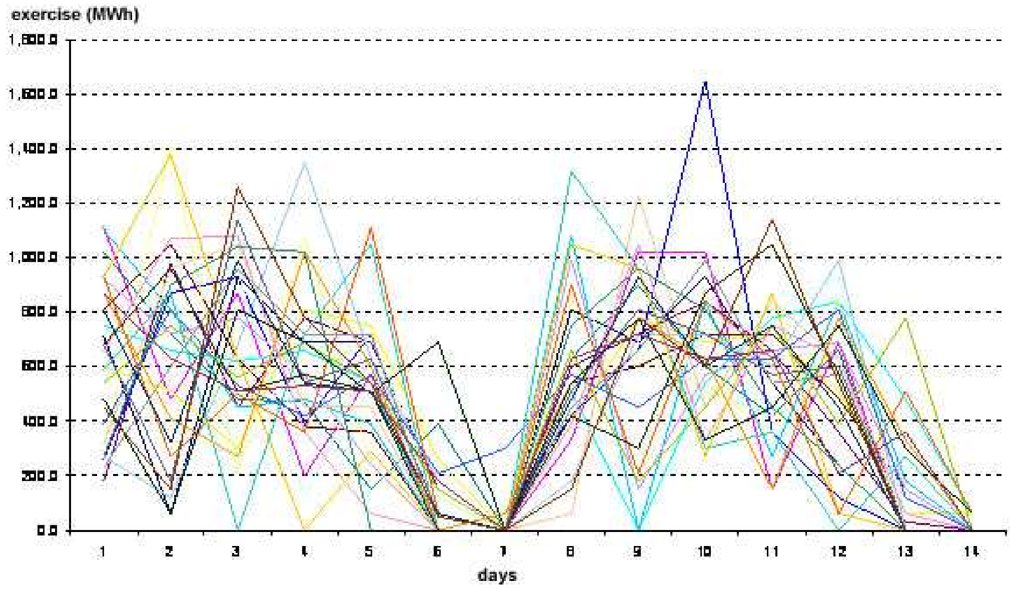

FIGURE 6. Daily exercises under the weekly constraint

TABLE 4. Mean loss (in %, over both quarters) due to subperiod constraints

0.4	fixed monthly amounts (deterministically optimal)
1.0	fixed weekly amounts (deterministically optimal)
4.2	identical hourly amounts (deterministic optimization: case 1)

range because of the small auto-regression time lag (25 hours). Thus we can only expect a very moderate accuracy of the long-term approximation from the sampled scenarios.

Figure 6 shows the optimal daily exercise volumes (not the hourly volumes, for the sake of better visibility) under the weekly constraint for all 30 scenarios. These values vary considerably both within single scenarios during a week and between different scenarios at a given hour. The difference between workdays and weekend is quite apparent.

Figures 7–8 show the weekly profits under the weekly and the monthly constraint, respectively. It can be seen for the first quarter (which has a low coverage) how the monthly constraint results in a higher variance of the profit than the weekly one, due to potentially higher flexibility in exercising the contract.

Although these results are not statistically significant, the qualitative conclusions are supported by the quantitative results of experiment 3.

5.3. **Experiment 3.** In this experiment, the cumulated energy of every scenario on every subperiod (hour, week, or month) is fixed, matching the corresponding optimal amount of the deterministic optimization (experiment 1, case 1).

Table 4 presents the results for the mean loss with respect to the stochastic optimization on the scenario fan, i. e., case 3. The difference to Table 3 results from the fact that in experiment 2 we consider only the first quarter but here both quarters. Again, as in experiment 2, we see that the monthly constraint results in just 10% loss of the VSS (relative VSS = 4.4% at 50 MWh), and the weekly constraint results in less than 25%.

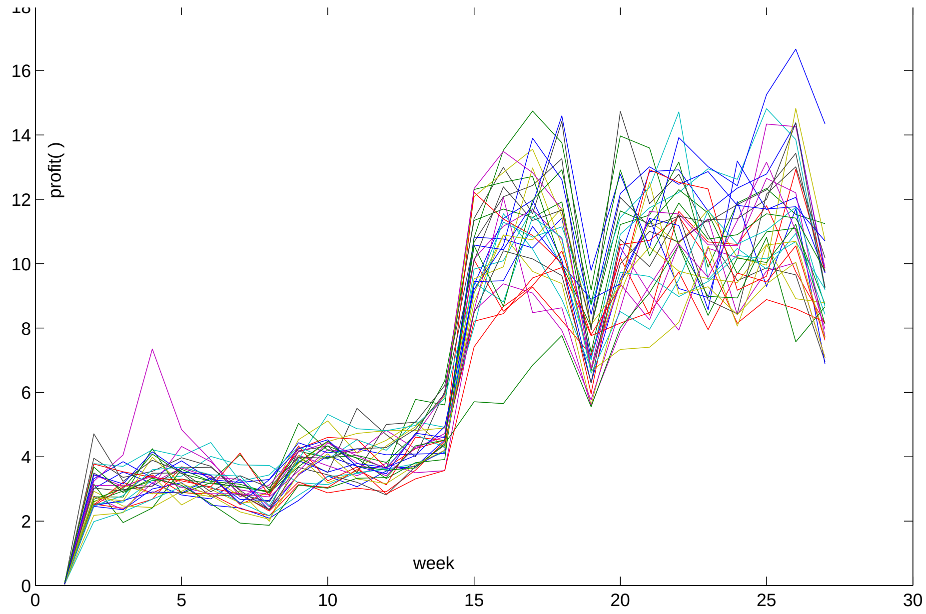

FIGURE 7. Weekly profits under the weekly constraint

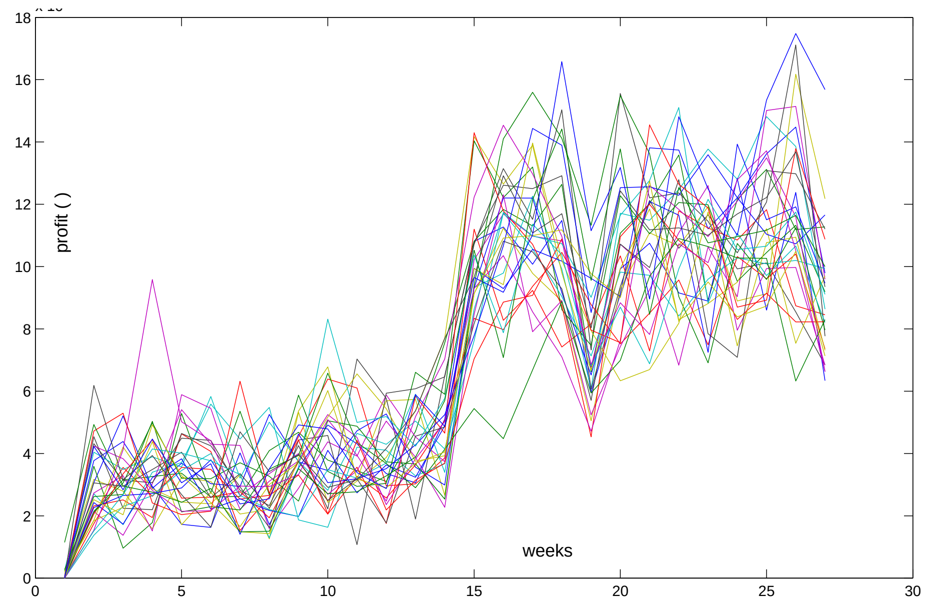

FIGURE 8. Weekly profits under the monthly constraint

6. DISCUSSION

The motivation for the investigations in this work was to analyze the gain of a stochastic optimization over the deterministic optimization (VSS) for a concrete valuation problem, in dependence on the flexibility for exercising the contract in its original form (experiment 1) or with additional subperiod constraints on the cumulated energy (experiments 2 and 3). The results reported in the last section encourage us to sketch in this section a heuristic for dimensioning and structuring the scenario trees for the specific problem of valuating swing options. This heuristic consists of the following steps:

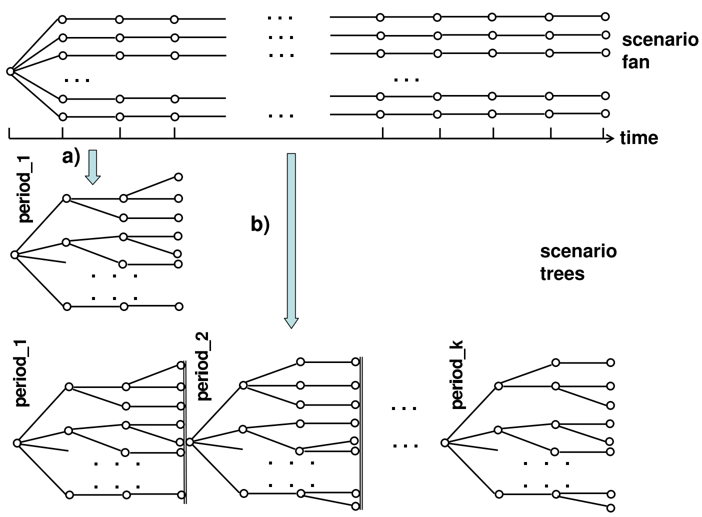

FIGURE 9. Steps 4a and b: reducing the scenario trees

(1) Determine the bounds on the optimal contract value, the lower bound given by the optimal value of the mean price scenario (expected value solution), and the upper bound given by the mean optimal value over all individual scenarios (wait-and-see solution).
(2) For various uniform subdivisions of the contract period with different values of the subperiod length, determine for each subinterval of every subdivision the cumulated exercise amount of the expected value solution.
(3) For every subperiod length, determine the mean loss of the optimal value with respect to the upper bound of step 1 if the subperiod exercise amounts of all scenarios are fixed to the values calculated in step 2.
(4) For a given approximation tolerance, select on the basis of step 3 the largest subperiod length whose mean loss remains within the tolerance. Now define approximate valuation problems using the selected subperiod length:
    a) For optimizing the exercise strategy over some initial period within the first subperiod (just the first day, say), use the valuation problem for the first subperiod with the cumulated exercise volume determined in step 2. This reduces the planning horizon; hence we can generate a scenario tree of substantially reduced complexity (by the methods cited in section 1, for instance). See Fig. 9a for an illustration of the principle.
    b) For valuating the contract over the entire planning horizon, the problem can be temporally decomposed to obtain independent subproblems over the selected subdivision. For each subperiod, a separate scenario tree can be generated, having as initial spot price the mean price over all scenarios and restricting the cumulated energy to the amount determined in step 2. Again, this will yield a substantial reduction of the problem complexity. See Fig. 9b for an illustration of the principle.

Next we discuss how the subproblems created by the temporal decomposition or reduction (as a special case) are related to the original problem and to each other. Of course, the selected approximation tolerance will not yield any hard bounds on how much one loses by the decomposition; rather, it is a measure how well the contract volume has been distributed over the subperiods. Given that the price model and the limited number of sampled scenarios will reflect the long-term behavior rather inaccurately in any case, we feel that the valuation error induced by the decomposition should be tolerable. It remains to discuss feasibility issues. Intertemporal dependencies of the decisions (exercises) in the original valuation problem (Sections 2–3) are introduced by

- the bounds on the cumulated exercise volume;
- the ratchets.

The first dependence is a long-term dependence, the second a short-term one. By introducing the subperiod constraints on cumulated exercise amounts as in experiment 3, we break the first dependence while keeping the total volume, thereby guaranteeing feasibility with respect to the cumulated energy of the original problem. The ramp constraints (ratchets) across subperiod boundaries, however, are dropped in step 4 of the temporal decomposition.

Now, when optimizing over the first subperiod (case 4a), the dropped ratchets belong to the following subperiod, and feasibility is still guaranteed with respect to all constraints of the original problem.

In case of step 4b, dropped ratchets belong to the beginning of every subperiod after the first, and will generally be violated. This can be prevented by additional constraints if desired. For instance, we could optimize the subproblems backward in time, recursively tightening exercise bounds in the last hour of the previous subperiod to force consistence with the ratchets relative to the optimal exercise in the first hour of the current subperiod. However, we are primarily interested in a good approximation of the contract volume, and in generating a good exercise strategy for some initial period. In practice the model will be reoptimized after that initial period, and the contract provides enough flexibility to generate a feasible exercise strategy this way. Therefore it is not necessary to enforce full feasibility in the decomposed problem.

In conclusion, the suggested heuristic promises the advantage of obtaining a good approximation of the contract value with substantially reduced effort and with easily parallelizable subproblems.

7. CONCLUSION

We have determined optimal values of an energy supply contract of swing option type for several stochastic and deterministic optimization models.

We have also calculated, with respect to a scenario fan, the expected value solution (given by deterministic optimization based on the mean price scenario), and the mean contract value under perfect information (given by averaging the optimal values of all individual scenarios). The two solutions provide lower and upper bounds, respectively, for the contract value obtained with any scenario tree constructed from the given fan by reduction techniques. The difference between the bounds equals the value of the stochastic solution (VSS) plus the negligible expected value of perfect information (EVPI).

By introducing additional subperiod constraints (thus fixing the exercise amounts of all scenarios to the corresponding optimal amounts of the deterministic optimization), we found that a smaller optimization horizon can be chosen without significant loss of the resulting contract value. This also allows for temporal decomposition of the valuation problem into independently solvable subperiod problems, yielding drastic reductions of the size of scenario trees. In combination with the fast solution algorithms presented in Sect. 2.3, we are thus able to solve good approximations of the valuation problem efficiently.

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